

DARK ENERGY AND DARK MATTER OF THE UNIVERSE FROM ONE-LOOP RENORMALIZATION OF RICCI

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Abstract

Here, creation of the universe is obtained only from gravity sector. The dynamical universe begins with two basic ingredients (i) vacuum energy, also called dark energy (as vacuum energy is not observed) and (ii) background radiation. These two are obtained through one-loop renormalization of riccion. Solutions of renormalization group equations yield initial value of vacuum energy with density $\rho_{\Lambda_{ew}} = 10^6 \text{GeV}^4$ as well as show a phase transition at the electroweak scale M_{ew} . As a result of phase transition, energy is released (in the form of background radiation) heating the universe upto temperature $T_{ew} = 78.5 \text{GeV} = 9.1 \times 10^{14} K$ initially. In the proposed cosmology, it is found that not only current universe expands with acceleration, but it undergoes accelerated expansion from the beginning itself. It is demonstrated that dark energy decays to dark matter as well as ratio of dark matter density and dark energy density remains less than unity upto a long

time in future universe also, providing a solution to *cosmic coincidence problem*. Future course of the universe is also discussed here. It is shown how entropy of the universe grows upto 10^{87} in the present universe. Moreover, particle creation, primordial nucleosynthesis and structure formation in the late universe is discussed for the proposed model. Thus, investigations, here, present a fresh look to cosmology consistent with current observational evidences as well as provide solution to some important problems.

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1.Introduction

Experimental probes, like luminosity measurements of high redshift supernova [1], anisotropies in cosmic microwave background [2] and observational gravitational clustering [3], strongly indicate late time accelerated expansion of the universe. Theoretically, accelerated expansion of the universe can be obtained either by modifying left hand side of Einstein's equations, like introduction of C-field in *steady-state theory* adhered to the Perfect Cosmological Principle [4] or by using energy momentum tensor, having dominance of *exotic* matter, with negative pressure violating the *strong energy condition*. This kind of matter is known as *dark energy* (vacuum energy), which has drawn much attention of cosmologists today. Past few years have witnessed

concerted efforts to propose different *dark energy* models. Important efforts, in this direction, are scalar field models like (i) quintessence [5], (ii) k-essence [6], (iii) tachyon scalar fields [7, 8, 9, 10, 11, 12] and models based on quantum particle production, Chaplygin gas [13, 14]. In these models, a scalar field acts as a source of *dark energy* and plays crucial role in the dynamical universe. But these models have *no* answer to the question “Where are these scalars coming from?” It is like Higg’s fields in GUTs as well as inflaton in inflationary models of the early universe. In the cosmology probed here, no scalar field is required to be incorporated, from outside the theory, to discuss cosmic *dark energy*. Following arguments in [14, 15], here also, vacuum energy is recognized as DE.

All these dark energy models (mentioned above) try to explain rolling down of DE density from a very high value in the early universe to an extremely small value in the current universe as it is suggested by astronomical observations. But its initial value, in the early universe, is different in different contexts. For example, it is $\sim 10^{76}\text{GeV}^4$ at Planck scale, $\sim 10^{60}\text{GeV}^4$ at GUT phase transition and quantum chromodynamics yields its value $\sim 10^{-3}\text{GeV}^4$. In what follows, one-loop renormalization of riccion (a particle representing physical role of the Ricci scalar explained below) contributes initial value equal to 10^6GeV^4 for slowly varying time-dependent DE density to the dynamical universe beginning with a phase transition at the electro-weak scale $M_{\text{ew}} = 100\text{GeV}$.

Work starts effectively from a general case of $(4 + D)$ -dimensional space-time and, later on, it is shown that $D \rightarrow 6$ making effective dimension of the space-time equal to 10. The observable universe is

a 4-dimensional hypersurface of the higher-dimensional world. So far sharpest experiments could probe gravity upto 0.1 mm.and, in the matter sector, probe could be possible upto electroweak scale $M_{\text{ew}} = 100\text{GeV}$, having wavelength as short as $\sim 1.97 \times 10^{-16}\text{cm}$. It means that it is difficult to realize 4-dimensional gravity for length scales less than 0.1 mm. So, it is reasonable to think higher-dimensional gravity for scales smaller than this scale. This idea is parallel to the brane-world gravity, where it is assumed that gravity is stronger in higher-dimensional space-time, called *bulk* and only a small part of it is realized in the observable universe [17].

It has been noted by many physicists that the *Ricci scalar* R behaves like a physical field also, in addition to its geometrical nature, if gravitational action contains higher-derivative terms [18, 19, 20, 21, 22, 23, 24, 25]. In 1980, Starobinsky suggested that if sign of the R^2 term in the higher-derivative gravitational action is chosen properly, one could obtain only one scalar particle with positive energy and positive squared mass. He called it as “scalaron”[18]. In papers [18, 19],gravitational constant G is either taken equal to unity or lagrangian density is taken as $\frac{1}{16\pi G}(R + \text{higher - derivative terms})$. As a result, $(\text{mass})^2$ of the Ricci scalar, does not depend on the gravitational constant G . Realizing the important role of G in gravity, in papers [20, 21, 22, 23, 24, 25] as well as here, lagrangian density is taken as $(\frac{1}{16\pi G}R + \text{higher - derivative terms})$ leading to a drastic change where $(\text{mass})^2$ depends on G also. In the following section as well as earlier works , it is shown that physical aspect of the *Ricci scalar* is given by a scalar field $\tilde{R} = \eta R$ (η is a parameter having length dimension), called *riccion* with $(\text{mass})^2$ depending on gravitational constant G and other coupling constants in

the action [20, 21, 22, 23, 24, 25]. It is different from *scalaron* in two ways (i) mass dimension of *riccion* is one like other scalar fields, such as quintessence, inflaton and Higg's scalar, whereas mass dimension of scalaron is two and (ii) $(mass)^2$ for scalaron does not depend on G , whereas, for a *riccion*, it depends on G .

The present paper begins with higher-derivative gravitational action in higher-dimensional space-time with topology $M^4 \otimes S^D$, where S^D is a D -dimensional sphere being the *hidden* extra-dimensional compact space. Higher-derivative gravity faces ghost problem, which can be avoided by taking coupling constants in the action properly.

The distance function for $(4 + D)$ -dimensional space-time is defined as

$$dS^2 = g_{\mu\nu} dx^\mu dx^\nu - l^2 d\Omega^2 \quad (1.1a)$$

with

$$d\Omega^2 = d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \cdots + \sin^2\theta_1 \cdots \sin^2\theta_{(D-1)} d\theta_D^2. \quad (1.1b)$$

Here $g_{\mu\nu}(\mu, \nu = 0, 1, 2, 3)$ are components of the metric tensor in M^4 , l is radius of the sphere which is independent of coordinates x^μ and $0 \leq \theta_1, \theta_2, \cdots, \theta_{(D-1)} \leq \pi$ and $0 \leq \theta_D \leq 2\pi$.

It is important to mention here that *riccion* is different from the scalar mode of *graviton*. which is highlighted in Appendix A.

Thus, in the proposed cosmological scenario, our dynamical universe begins with the initial *dark energy* density (vacuum energy density) $\rho_{\Lambda_{ew}} = 1.16 \times 10^7 \text{GeV}^4$ and background radiation with temperature $T_{ew} = 33.5 \text{GeV} = 3.89 \times 10^{14} \text{K}$, contributed by *riccion* at M_{ew} . The background radiation is caused by a phase transition at M_{ew} . This event

is recognized as *big – bang*. As usual, dynamical universe grows in the *post big – bang* era. In the present theory, *riccion* has effective role in the *pre big – bang* era, but it remains *passive* in the *post big – bang* times.

The paper is organized as follows. Section 2 demonstrates derivation of *riccion* equation and its action. One-loop renormalization of *riccion* is done in section 3. Renormalization group equations are solved in section 4. This is an important section as it provides initial values of *dark energy* density ρ_Λ and temperature T of the observable universe. Moreover, in this section, dimension of *hidden* space is derived to be 6. In section 5, derivation of equation of state for *dark energy*(vacuum energy) is derived and phase transition at M_{ew} is discussed. It is found that *dark energy* decreases with expansion of the universe. So, it is natural to think for decay of *dark energy* to *dark matter*, which is discussed in section 6. This section is very important from cosmological point of view, as many important results are derived here. It is demonstrated that *dark energy* decays to *hot dark matter* (HDM) till matter remains in thermal equilibrium with radiation, but when temperature falls down the decoupling temperature, production of HDM decreases and creation of *cold dark matter* (CDM) increases. *Dark matter* density is found less than *dark energy* density from the epoch of *big – bang* upto the time $10.6t_0$ (t_0 is the present age of the universe). This result provides a solution to *cosmic coincidence problem*, which raises the question “ Why does *dark energy* dominates over *dark matter* only recently?.” First time, this question was posed by P.J.Steihardt [26]. In some earlier works also, this problem was addressed and solutions were suggested, taking coupled system of *quintessence* scalar

fields and matter [27, 29, 30, 31]. As mentioned above, contrary to earlier attempts, such scalar fields are not required here. Moreover, it is found that the universe undergoes an accelerated expansion from the beginning itself upto late future universe. Future course of dynamics is also discussed here. In this section, temperature and entropy of the universe are discussed and it is demonstrated how entropy of the universe grows upto 10^{87} upto the current epoch. In section 7, creation of spinless and spin-1/2 particles, primordial nucleosynthesis as well as structure formation in the late universe are discussed. The last section summarizes results.

Natural units, defined as $\kappa_B = \hbar = c = 1$ (where κ_B is Boltzman's constant, \hbar is Planck's constant divided by 2π and c is the speed of light) are used here with GeV is used as a fundamental unit such that $1\text{GeV} = 1.16 \times 10^{13} K = 1.78 \times 10^{-24} gm$, $1\text{GeV}^{-1} = 1.97 \times 10^{-14} cm = 6.58 \times 10^{-25} sec$.

2. Riccions from (4+D)-dimensional geometry of the space-time

Theory begins with the gravitational action

$$S = \int d^4x d^Dy \sqrt{-g_{(4+D)}} \left[\frac{M^{(2+D)} R_{(4+D)}}{16\pi} + \alpha_{(4+D)} R_{(4+D)}^2 + \gamma_{(4+D)} (R_{(4+D)}^3 - \frac{6(D+3)}{(D-2)} \square_{(D+4)} R_{(D+4)}^2) \right], \quad (2.1a)$$

where $G_{(4+D)} = M^{-(2+D)}$ (M being the mass scale), $\alpha_{(4+D)} = \alpha V_D^{-1}$, $\gamma_{(4+D)} = \frac{\eta^2}{3!(D-2)} V_D^{-1}$, and $R_D = \frac{D(D-1)}{l^2}$. Here V_D , being the volume of S^D , is given as

$$V_D = \frac{2\pi^{(D+1)/2}}{\Gamma(D+1)/2} l^D. \quad (2.1b)$$

$g_{(4+D)}$ is determinant of the metric tensor g_{MN} ($M, N = 0, 1, 2, \dots, (3+D)$) and $R_{(4+D)} = R + R_D$. α is a dimensionless coupling constant, R is the Ricci scalar in M^4 and $G_N = G_{(4+D)}/V_D$.

Invariance of S under transformations $g_{MN} \rightarrow g_{MN} + \delta g_{MN}$ yields [24, 25]

$$\frac{M^{(2+D)}}{16\pi} (R_{MN} - \frac{1}{2} g_{MN} R_{(4+D)}) + \alpha_{(4+D)} H_{MN}^{(1)} + \gamma_{(4+D)} H_{MN}^{(2)} = 0, \quad (2.2a)$$

where

$$H_{MN}^{(1)} = 2R_{;MN} - 2g_{MN} \square_{(4+D)} R_{(4+D)} - \frac{1}{2} g_{MN} R_{(4+D)}^2 + 2R_{(4+D)} R_{MN}, \quad (2.2b)$$

and

$$H_{MN}^{(2)} = 3R_{;MN}^2 - 3g_{MN} \square_{(4+D)} R_{(4+D)}^2 - \frac{6(D+3)}{(D-2)} \left\{ -\frac{1}{2} g_{MN} \square_{(4+D)} R_{(4+D)}^2 + 2\square_{(4+D)} R_{(D+4)} R_{MN} + R_{;MN}^2 \right\} - \frac{1}{2} g_{MN} R_{(4+D)}^3 + 3R_{(4+D)}^2 R_{MN} \quad (2.2c)$$

with semi-colon (;) denoting curved space covariant derivative and

$$\square_{(4+D)} = \frac{1}{\sqrt{-g_{(4+D)}}} \frac{\partial}{\partial x^M} \left(\sqrt{-g_{(4+D)}} g^{MN} \frac{\partial}{\partial x^N} \right)$$

Trace of these field equations is obtained as

$$-\left[\frac{(D+2)M^{(2+D)}}{32\pi} \right] R_{(4+D)} + \alpha_{(4+D)} [2(D+3) \square_{(4+D)} R_{(4+D)} + \frac{1}{2} D R_{(4+D)}^2] + \frac{1}{2} \gamma_{(4+D)} (D-2) R_{(4+D)}^3 = 0. \quad (2.3)$$

In the space-time described by the distance function defined in eq.(1.1),

$$\square_{(4+D)} R_{(4+D)} = \square R = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left(\sqrt{-g} g^{\mu\nu} \frac{\partial}{\partial x^\nu} \right) R, \quad (2.4)$$

using the definition of $R_{(4+D)}$ given in eq.(2.1).

Connecting eqs.(2.3)- (2.4) as well as using $R_{(4+D)}$, $\alpha_{(4+D)}$ and $\gamma_{(4+D)}$ from eq.(2.1), one obtains in M^4

$$-\left[\frac{(D+2)M^{(2+D)}V_D}{32\pi}\right](R+R_D)+\alpha[2(D+3)\Box R + \frac{1}{2}D(R+R_D)^2] + \frac{\eta^2}{12}(R+R_D)^3 = 0, \quad (2.5)$$

which is re-written as

$$[\Box + \frac{1}{2}\xi R + m^2 + \frac{\lambda}{3!}\eta^2 R^2]R + \eta^{-1}\vartheta = 0, \quad (2.6)$$

where

$$\begin{aligned} \xi &= \frac{D}{2(D+3)} + \eta^2 \lambda R_D \\ m^2 &= -\frac{(D+2)\lambda M^{(2+D)}V_D}{16\pi} + \frac{DR_D}{2(D+3)} + \frac{1}{2}\eta^2 \lambda R_D^2 \\ \lambda &= \frac{1}{4(D+3)\alpha}, \\ \vartheta &= \eta \left[-\frac{(D+2)\lambda M^{(2+D)}V_D}{16\pi} + \frac{DR_D^2}{4(D+3)} + \frac{1}{6}\eta^2 \lambda R_D^3 \right], \end{aligned} \quad (2.7a, b, c, d)$$

where $\alpha > 0$ to avoid the ghost problem.

A scalar field, representing a spinless particle, has unit mass dimension in existing theories. R , being combination of second order derivative as well as squares of first order derivative of metric tensor components with respect to space-time coordinates, has mass dimension 2. So, to have mass dimension like other scalar fields, eq.(2.6) is multiplied by η and ηR is recognized as \tilde{R} . As a result, this equation looks like

$$[\square + \frac{1}{2}\xi R + m^2 + \frac{\lambda}{3!}\tilde{R}^2]\tilde{R} + \vartheta = 0. \quad (2.8)$$

For *scalaron*, η is dimensionless [18].

The above analyses show that, on taking trace of field equations (2.2) and compactifying the space-time $M^4 \otimes S^D$ to M^4 , *only one degree of freedom* is obtained spontaneously, which is the scalar mode \tilde{R} [16, 18-23]. It is unlike *gravitons* having 5 degrees of freedom including one scalar. In Appendix A, it is explained that scalar mode of *graviton* is *different* from *riccion*.

If \tilde{R} is a basic physical field, there should be an action $S_{\tilde{R}}$ yielding eq.(2.8) for invariance of $S_{\tilde{R}}$, under transformations $\tilde{R} \rightarrow \tilde{R} + \delta\tilde{R}$.

In what follows, $S_{\tilde{R}}$ is obtained. If such an action exists, one can write

$$\delta S_{\tilde{R}} = - \int d^4x \sqrt{-g} \delta\tilde{R} [\square + \frac{1}{2}\xi R + m^2 + \frac{\lambda}{3!}\tilde{R}^2]\tilde{R} + \vartheta] \quad (2.9a)$$

which yields eq.(2.8) if $\delta S_{\tilde{R}} = 0$ under transformations $\tilde{R} \rightarrow \tilde{R} + \delta\tilde{R}$.

Eq.(2.9a) is re-written as

$$\begin{aligned} \delta S_{\tilde{R}} &= \int d^4x \sqrt{-g} \left[\partial^\mu \tilde{R} \partial_\mu (\delta\tilde{R}) - \left(\frac{1}{2}\xi R \tilde{R}^2 + m^2 \tilde{R} + \frac{\lambda}{3!} \tilde{R}^3 + \vartheta \right) \delta\tilde{R} \right] \\ &= \int d^4x \delta \left\{ \sqrt{-g} \left[\frac{1}{2} \partial^\mu \tilde{R} \partial_\mu \tilde{R} - \left(\frac{1}{3!} \xi R \tilde{R}^2 + \frac{1}{2} m^2 \tilde{R}^2 + \frac{\lambda}{4!} \tilde{R}^4 + \vartheta \tilde{R} \right) \right] \right\}. \end{aligned} \quad (2.9b)$$

R, \tilde{R} and $d^4x \sqrt{-g}$ are invariant under co-ordinate transformations. So, $R(x) = R(X), \tilde{R}(x) = \tilde{R}(X)$ and $d^4x \sqrt{-g} = d^4X$, where $X^i (i = 0, 1, 2, 3)$ are local and $x^i (i = 0, 1, 2, 3)$ are global coordinates. Moreover,

$$\square = g^{ij} \frac{\partial^2}{\partial x^i \partial x^j} + \frac{1}{2} g^{mn} \frac{\partial g_{mn}}{\partial x^i} g^{ij} \frac{\partial}{\partial x^j} + \frac{\partial g^{ij}}{\partial x^i} \frac{\partial}{\partial x^j} = \frac{\partial^2}{\partial X^i \partial X^j}$$

in a locally inertial co-ordinate system, where $g_{ij} = \eta_{ij}$ (components of Minkowskian metric) and $g_{,i}^{ij} = 0$ (comma (,) stands for partial derivative). Thus, in a locally inertial co-ordinate system,

$$\begin{aligned}\delta S_{\tilde{R}} &= \int d^4x \delta \left\{ \sqrt{-g} \left[\frac{1}{2} \partial^\mu \tilde{R} \partial_\mu \tilde{R} - \left(\frac{1}{3!} \xi R \tilde{R}^2 + \frac{1}{2} m^2 \tilde{R}^2 + \frac{\lambda}{4!} \tilde{R}^4 + \vartheta \tilde{R} \right) \right] \right\} \\ &= \int d^4X \quad \delta \left\{ \sqrt{-g} \left[\frac{1}{2} \partial^\mu \tilde{R} \partial_\mu \tilde{R} - \left(\frac{1}{3!} \xi R \tilde{R}^2 + \frac{1}{2} m^2 \tilde{R}^2 + \frac{\lambda}{4!} \tilde{R}^4 + \vartheta \tilde{R} \right) \right] \right\} \\ &= \delta \int d^4X \quad \left\{ \sqrt{-g} \left[\frac{1}{2} \partial^\mu \tilde{R} \partial_\mu \tilde{R} - \left(\frac{1}{3!} \xi R \tilde{R}^2 + \frac{1}{2} m^2 \tilde{R}^2 + \frac{\lambda}{4!} \tilde{R}^4 + \vartheta \tilde{R} \right) \right] \right\}\end{aligned}$$

Employing principles of covariance and equivalence as well as eq.(2.9b)

$$\delta S_{\tilde{R}} = \delta \int d^4x \left\{ \sqrt{-g} \left[\frac{1}{2} \partial^\mu \tilde{R} \partial_\mu \tilde{R} - \left(\frac{1}{3!} \xi R \tilde{R}^2 + \frac{1}{2} m^2 \tilde{R}^2 + \frac{\lambda}{4!} \tilde{R}^4 + \vartheta \tilde{R} \right) \right] \right\},$$

which implies that

$$S_{\tilde{R}} = \int d^4x \left\{ \sqrt{-g} \left[\frac{1}{2} \partial^\mu \tilde{R} \partial_\mu \tilde{R} - \left(\frac{1}{3!} \xi R \tilde{R}^2 + \frac{1}{2} m^2 \tilde{R}^2 + \frac{\lambda}{4!} \tilde{R}^4 + \vartheta \tilde{R} \right) \right] \right\}. \quad (2.10)$$

It is important to mention here that \tilde{R} is different from other scalar fields due to dependence of (mass)² on the gravitational constant, dimensionality of the space-time and the coupling constant α , given by the eq.(2.7b). Moreover, it emerges from geometry of the space-time.

3.One-loop quantum correction and renormalization of riccion

The $S_{\tilde{R}}$ with the lagrangian density, given by eq.(2.10), can be expanded around the classical minimum \tilde{R}_0 in powers of quantum fluctuation $\tilde{R}_q = \tilde{R} - \tilde{R}_0$ as

$$S_{\tilde{R}} = S_{\tilde{R}}^{(0)} + S_{\tilde{R}}^{(1)} + S_{\tilde{R}}^{(2)} + \cdots,$$

where

$$\begin{aligned} S_{\tilde{R}}^{(0)} &= \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \tilde{R}_0 \partial_\nu \tilde{R}_0 - \left(\frac{1}{3!} \xi \tilde{R}_0^3 + \frac{1}{2} m^2 \tilde{R}_0^2 + \frac{\lambda}{4!} \tilde{R}_0^4 + \vartheta \tilde{R}_0 \right) \right] \\ S_{\tilde{R}}^{(2)} &= \int d^4x \sqrt{-g} \tilde{R}_q \left[\square + \frac{1}{2} \xi R + m^2 + \frac{\lambda}{2!} \tilde{R}_0^2 \right] \tilde{R}_q \end{aligned}$$

and

$$S_{\tilde{R}}^{(1)} = 0$$

as usual, because this term contains the classical equation.

The effective action is expanded in powers of \hbar (with $\hbar = 1$) as

$$\Gamma(\tilde{R}) = S_{\tilde{R}} + \Gamma^{(1)} + \Gamma'$$

with one-loop correction given as [32, 33]

$$\Gamma^{(1)} = \frac{i}{2} \ln \text{Det}(D/\mu^2), \quad (3.1a)$$

where

$$D \equiv \frac{\delta^2 S_{\tilde{R}}}{\delta \tilde{R}^2} \Big|_{\tilde{R}=\tilde{R}_0} = \square + \frac{1}{2} \xi R + m^2 + \frac{\lambda}{2!} \tilde{R}_0^2 \quad (3.1b)$$

and Γ' is a term for higher-loop quantum corrections. In eq.(3.1), μ is a mass parameter to keep $\Gamma^{(1)}$ dimensionless.

To evaluate $\Gamma^{(1)}$, the operator regularization method [34] is used upto adiabatic order 4. As potentially divergent terms are expected upto this order only. In a 4-dim. theory, one-loop correction is obtained as

$$\begin{aligned} \Gamma^{(1)} &= (16\pi^2)^{-1} \frac{d}{ds} \left[\int d^4x \sqrt{-g(x)} \left(\frac{\tilde{M}^2}{\mu^2} \right)^{-s} \left\{ \frac{\tilde{M}^4}{(s-2)(s-1)} \right. \right. \\ &\quad + \frac{\tilde{M}^2}{(s-1)} \left(\frac{1}{6} - \frac{1}{2} \xi \right) R + \left[\frac{1}{6} \left(\frac{1}{5} - \frac{1}{2} \xi \right) \square R + \frac{1}{180} R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} \right. \\ &\quad \left. \left. - \frac{1}{180} R^{\mu\nu} R_{\mu\nu} + \frac{1}{2} \left(\frac{1}{6} - \frac{1}{2} \xi \right)^2 R^2 \right] \right\} \Big|_{s=0}, \end{aligned}$$

$$(3.2a)$$

where

$$\tilde{M}^2 = m^2 + (\lambda/2)\tilde{R}_0^2. \quad (3.2b)$$

Here it is important to note that matter as well as geometrical both aspects of the Ricci scalar are used in eq.(3.2). The matter aspect is manifested by \tilde{R} and the geometrical aspect by R , Ricci tensor components $R_{\mu\nu}$ and curvature tensor components $R_{\mu\nu\alpha\beta}$ as it is mentioned above also.

After some manipulations, the lagrangian density in $\Gamma^{(1)}$ is obtained as

$$\begin{aligned} L_{\Gamma^{(1)}} = & (16\pi^2)^{-1} \left[(m^2 + (\lambda/2)\tilde{R}_0^2)^2 \left\{ \frac{3}{4} - \frac{1}{2} \ln \left(\frac{m^2 + (\lambda/2)\tilde{R}_0^2}{\mu^2} \right) \right\} \right. \\ & - \left(\frac{1}{6} - \frac{1}{2}\xi \right) R (m^2 + (\lambda/2)\tilde{R}_0^2) \left\{ 1 - \ln \left(\frac{m^2 + (\lambda/2)\tilde{R}_0^2}{\mu^2} \right) \right\} \\ & - \ln \left(\frac{m^2 + (\lambda/2)\tilde{R}_0^2}{\mu^2} \right) \left\{ \frac{1}{6} \left(\frac{1}{5} - \frac{1}{2}\xi \right) \square R + \frac{1}{180} R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} \right. \\ & \left. \left. - \frac{1}{180} R^{\mu\nu} R_{\mu\nu} + \frac{1}{2} \left(\frac{1}{6} - \frac{1}{2}\xi \right)^2 R^2 \right\} \right]. \end{aligned} \quad (3.3)$$

Now the renormalized form of lagrangian density can be written as

$$\begin{aligned} L_{\text{ren}} = & \frac{1}{2} g^{\mu\nu} \partial_\mu \tilde{R}_0 \partial_\nu \tilde{R}_0 - \frac{\xi}{3!\eta} \tilde{R}_0^3 - \frac{1}{2} m^2 \tilde{R}_0^2 - \frac{\lambda}{4!} \tilde{R}_0^4 - \vartheta \tilde{R}_0 + \tilde{\Lambda} \\ & + \epsilon_0 R + \frac{1}{2} \epsilon_1 R^2 + \epsilon_2 R^{\mu\nu} R_{\mu\nu} + \epsilon_3 R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} \\ & + \epsilon_4 \square R + L_{\Gamma^{(1)}} + L_{\text{ct}} \end{aligned} \quad (3.4a)$$

with bare coupling constants $\lambda_i \equiv (m^2, \vartheta, \lambda, \tilde{\Lambda}, \xi, \epsilon_0, \epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4), \Gamma^{(1)}$ given by eq.(3.3) and L_{ct} given as

$$\begin{aligned}
 L_{\text{ct}} = & -\frac{1}{2}\delta\xi R\tilde{R}_0^2 - \frac{1}{2}\delta m^2 \tilde{R}_0^2 - \frac{\delta\lambda}{4!}\tilde{R}_0^4 - \delta\vartheta\tilde{R}_0 + \delta\tilde{\Lambda} + \delta\epsilon_0 R + \frac{1}{2}\delta\epsilon_1 R^2 \\
 & + \delta\epsilon_2 R^{\mu\nu} R_{\mu\nu} + \delta\epsilon_3 R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} + \delta\epsilon_4 \square R.
 \end{aligned}
 \tag{3.4b}$$

In eq.(3.4b), $\delta\lambda_i \equiv (\delta m^2, \delta\vartheta, \delta\lambda, \delta\tilde{\Lambda}, \delta\xi, \delta\epsilon_0, \delta\epsilon_1, \delta\epsilon_2, \delta\epsilon_3, \delta\epsilon_4)$ are counter-terms, which are calculated using the following renormalization conditions [35, 36]

$$\begin{aligned}
 \tilde{\Lambda} &= L_{\text{ren}}|_{\tilde{R}_0=\tilde{R}_{(0)0}, R=0} \\
 \lambda &= -\frac{\partial^4}{\partial\tilde{R}_0^4} L_{\text{ren}}|_{\tilde{R}_0} = \tilde{R}_{(0)1, R=0} \\
 \vartheta &= -\frac{\partial}{\partial\tilde{R}_0} L_{\text{ren}}|_{\tilde{R}_0=\tilde{R}_{(0)1}, R=0} \\
 m^2 &= -\frac{\partial^2}{\partial\tilde{R}_0^2} L_{\text{ren}}|_{\tilde{R}_0=0, R=0} \\
 \frac{1}{2}\xi &= -\eta \frac{\partial^3}{\partial R \partial \tilde{R}_0^2} L_{\text{ren}}|_{\tilde{R}_0=\tilde{R}_{(0)2}, R=0} \\
 \epsilon_0 &= \frac{\partial}{\partial R} L_{\text{ren}}|_{\tilde{R}_0=0, R=0} \\
 \epsilon_1 &= \frac{\partial^2}{\partial R^2} L_{\text{ren}}|_{\tilde{R}_0=0, R=R_5} \\
 \epsilon_2 &= \frac{\partial}{\partial(R^{\mu\nu} R_{\mu\nu})} L_{\text{ren}}|_{\tilde{R}_0=0, R=R_6} \\
 \epsilon_3 &= \frac{\partial}{\partial(R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta})} L_{\text{ren}}|_{\tilde{R}_0=0, R=R_7} \\
 \epsilon_4 &= \frac{\partial}{\partial(\square R)} L_{\text{ren}}|_{\tilde{R}_0=0, R=R_8}.
 \end{aligned}
 \tag{3.5a, b, c, d, e, f, g, h, i, j}$$

As $\tilde{R} = \eta R$, so when $R = 0$, $\tilde{R}_{(0)0} = \tilde{R}_{(0)1} = \tilde{R}_{(0)2} = 0$ and $R_5 = R_6 = R_7 = R_8 = 0$ when $\tilde{R}_0 = 0$.

4. Renormalization group equations and their solutions

Mass scale dependence of coupling constants are obtained by solving renormalization group equations

$$\frac{d\lambda_i}{d\tau} = \beta_{\lambda_i}, \quad (4.1)$$

where $\tau = \frac{1}{2} \ln(M_{\text{ew}}^2/\mu^2)$ and β_{λ_i} are one-loop β functions for different coupling constants. Here μ is the mass scale parameter and M_{ew} is the cut-off scale such that $\mu \geq M_{\text{ew}}$. As experimental probes could be possible upto M_{ew} , so this scale is used as a cut-off mass scale.

β_{λ_i} in eq.(4.1) are calculated using counter-terms yielded by eqs.(3.4) and (3.5) and putting $\mu \frac{d}{d\mu} \lambda_i = 0$ for bare coupling constants in equations

$$\beta_{\lambda_i} = \mu \frac{d}{d\mu} (\lambda_i + \delta \lambda_i) \Big|_{\lambda_i} \quad (4.2)$$

Using β -functions for different coupling constants, given by eqs.(4.2), solutions of differential equations (4.1) are derived, as

$$\begin{aligned} \tilde{\Lambda} &= \tilde{\Lambda}_{\text{ew}} + \frac{m_{\text{ew}}^4}{2\lambda_{\text{ew}}} \left[\left(1 - \frac{3\lambda_{\text{ew}}\tau}{8\pi^2} \right)^{1/3} - 1 \right] \\ \lambda &= \lambda_{\text{ew}} \left[1 - \frac{3\lambda_{\text{ew}}\tau}{8\pi^2} \right]^{-1} \\ \vartheta &= \vartheta_{\text{ew}}(\text{constant}) \\ m^2 &= m_{\text{ew}}^2 \left[1 - \frac{3\lambda_{\text{ew}}\tau}{8\pi^2} \right]^{-1/3} \\ \frac{1}{2}\xi &= \frac{1}{6} + \left(\frac{1}{2}\xi_{\text{ew}} - \frac{1}{6} \right) \left[1 - \frac{3\lambda_{\text{ew}}\tau}{8\pi^2} \right]^{-1} \end{aligned} \quad (4.3a, b, c, d, e)$$

for coupling constants of relevant terms to be used in further investigations in the present paper. Here, $\lambda_{i_{\text{ew}}} = \lambda_i(\tau = 0)$ and $\tau = 0$ at $\mu = M_{\text{ew}}$ according to definition of τ given above.

These results show that as $\mu \rightarrow \infty(\tau \rightarrow -\infty)$, $\lambda \rightarrow 0$ and $m^2 \rightarrow 0$ and $\frac{1}{2}\xi \rightarrow \frac{1}{6}$.

Using these limits in eq.(2.7a), it is obtained that

$$D = 6. \quad (4.4)$$

Also,eqs.(2.1a) and (4.4) imply

$$R_6 = \frac{30}{l^2}. \quad (4.5)$$

So, from eq.(4.3a)

$$\tilde{\Lambda} = \tilde{\Lambda}_{\text{ew}} \quad (4.6)$$

at $\mu = M_{\text{ew}}$. The equation (4.3c) shows that ϑ is independent of mass scale μ . So, $\eta^{-1}\vartheta$, being true for all μ , is obtained from eq.(2.7d) as

$$\eta^{-1}\vartheta = \frac{30}{l_{\text{ew}}^2} \left[m_{\text{ew}}^2 - \frac{5(1 + 60\lambda_{\text{ew}})}{l_{\text{ew}}^2} \right], \quad (4.6)$$

at $\mu = M_{\text{ew}}$. Here

$$m_{\text{ew}}^2 = -\frac{\lambda_{\text{ew}} M_{\text{ew}}^8 V_{6\text{ew}}}{2\pi} + \frac{10(1 + 450\lambda_{\text{ew}})}{l_{\text{ew}}^2} \quad (4.7)$$

with

$$V_{6\text{ew}} = \frac{16\pi^3 l_{\text{ew}}^6}{15} \quad (4.8)$$

which is derived connecting eqs.(2.7b,d), taking the arbitrary parameter $\eta = l_{\text{ew}}$ and putting $D = 6$.

Thus $\eta^{-1}\vartheta$, given by eq.(4.6), is an imprint of extra six-dimensional compact manifold S^6 in the 4-dimensional universe. This term has the dimension of energy density. Moreover, it is not generated through

matter, but the geometry of extra-dimensional space. So, it is recognized as *dark energy* density $\rho_{\Lambda_{\text{ew}}}$ at $\mu = M_{\text{ew}}$, given as

$$\rho_{\Lambda_{\text{ew}}} = \eta^{-1} \vartheta = \frac{30}{l_{\text{ew}}^2} \left[m_{\text{ew}}^2 - \frac{5(1 + 60\lambda_{\text{ew}})}{l_{\text{ew}}^2} \right], \quad (4.9)$$

5. Equation of state for dark energy and Phase transition at the eletroweak scale

Dark energy density $\rho_{\Lambda_{\text{ew}}}$, obtained through renormalization of riccion at energy mass scale $\mu = M_{\text{ew}}$, is given by eq.(4.9). As riccion is emerging from geometry of the space-time, $\rho_{\Lambda_{\text{ew}}}$ can be obtained as zero-point energy of riccion also, given as

$$\rho_{\Lambda}(t=0) = \rho_{\Lambda_{\text{ew}}} = (2\pi)^{-3} \int_0^k \sqrt{k^2 + m_{\text{ew}}^2} 4\pi k^2 dk, \quad (5.1)$$

where m_{ew} is the mass of riccion at the electroweak scale M_{ew} . Eq.(5.1) shows that $\rho_{\Lambda_{\text{ew}}}$ diverges as $k \rightarrow \infty$. But $\rho_{\Lambda_{\text{ew}}} = \tilde{\Lambda}_{\text{ew}}$ (as given by eq.(4.7)), being finite, implies that the integral in eq.(5.1) should be regularized upto a certain cut-off mode $k = k_c$. As so far experiments could be performed upto M_{ew} only, so cut-off scale is taken as $k_c = M_{\text{ew}}$ as above. Moreover, at this scale riccions heavier than M_{ew} can not survive, so $m_{\text{ew}} \leq M_{\text{ew}}$. Now eq.(5.1) yields

$$\rho_{\Lambda_{\text{ew}}} = \frac{\pi}{4(2\pi)^3} M_{\text{ew}}^4 [3\sqrt{2} - \ln(1 + \sqrt{2})] \quad (5.2)$$

taking $m_{\text{ew}} = M_{\text{ew}}$. The second quantization and uncertainty relation imply that vacuum has energy density as well as pressure [13]. Experimental probes, like Ia supernova and WMAP [1, 2, 3, 37] suggest accelerated expansion of the universe, which requires negative pressure for the vacuum. So, to have consistency with recent probes, the isotropic vacuum pressure is calculated as

$$p_{\Lambda_{\text{ew}}} = -\frac{(2\pi)^{-3}}{3} \int_0^{m_{\text{ew}}} 4\pi k^3 dk = -\frac{\pi}{3(2\pi)^3} M_{\text{ew}}^4. \quad (5.3)$$

It yields

$$p_{\Lambda_{\text{ew}}}/\rho_{\Lambda_{\text{ew}}} = \omega_{\Lambda_{\text{ew}}} = -\frac{4}{3}[3\sqrt{2}-\ln(1+\sqrt{2})]^{-1} = -0.397 \simeq -0.4. \quad (5.4a)$$

Though in certain models, time-dependence of $\omega = p/\rho$ is also proposed, but normally it is taken as a constant. So, here also, this ratio is considered independent of time. As a result

$$\omega_{\Lambda} = p_{\Lambda}/\rho_{\Lambda} = p_{\Lambda_{\text{ew}}}/\rho_{\Lambda_{\text{ew}}} = -0.4. \quad (5.4b)$$

Using $l_{\text{ew}}^{-1} = M_{\text{ew}} = m_{\text{ew}}$ and connecting eqs.(4.7),(4.8) and (5.2), a quadratic equation for λ_{ew} is obtained as

$$\left(450 - \frac{8\pi^2}{15}\right)\lambda_{\text{ew}}^2 + \left[20\left(450 - \frac{8\pi^2}{15}\right) + 30 \times 2818.8\right]\lambda_{\text{ew}} + 11375.3 = 0, \quad (5.5)$$

which is solved to

$$\lambda_{\text{ew}} = -0.013352958. \quad (5.6)$$

Also using $M_{\text{ew}} = m_{\text{ew}}$ in eq.(5.2), it is obtained that

$$\rho_{\Lambda_{\text{ew}}} \simeq 10^6 \text{GeV}^4. \quad (5.7)$$

Planck scale is supposed to be a fundamental scale in field theories. So, it is proposed that energy mass scale μ falls from the Planck mass $M_P = 10^{19} \text{GeV}$ to the cut-off scale $M_{\text{ew}} = 100 \text{GeV}$. When it happens so, phase transition takes place at $\mu = M_{\text{ew}}$ and energy with density

$$\rho_{\text{ew}(r)} = \tilde{\Lambda} - \tilde{\Lambda}_{\text{ew}} = 2.5 \times 10^7 \text{GeV}^4 \quad (5.8a)$$

is released, which is obtained connecting eqs.(4.3a) and (5.6).

In the proposed model (PM), this event is recognized as *big – bang*, being beginning of the universe like standard model(SMU) with the release of *background radiation* having energy density $\rho_{ew(r)}$, given by eq.(5.8a). In contrast to SMU, here, released energy density at the epoch of big-bang is finite (it is infinite in SMU). Temperature of photons T_{ew} with energy density ρ_{ew} , is obtained from

$$\rho_{ew(r)} = \frac{\pi^2}{15} T_{ew}^4 = 2.5 \times 10^7 \text{GeV}^4 \quad (5.8b)$$

as

$$T_{ew} = 78.5 \text{GeV} = 9.1 \times 10^{14} K. \quad (5.9)$$

6.Proposed cosmological scenario, Dark energy and Dark matter

In the standard model of the *big – bang* theory, it is supposed that, around 13.7Gyrs ago, there used to be a *fireball* (an extremely hot object), which was termed as *primeval atom* by Lemaître. Our universe came into existence, when this *primeval atom* burst out. This event is called *big – bang*.

According to the proposed cosmological picture, the main content of the *primeval atom* was *riccions*, being contribution of 10-dimensional higher-derivative gravity to the 4-dimensional world. One-loop renormalization of *riccions* and solutions of resulting group equations yield that when energy mass scale comes down to electroweak scale $M_{ew} = 100 \text{GeV}$, *riccion* contributes *dark energy* density $\rho_{\Lambda_{ew}} = 10^6 \text{GeV}^4$. Moreover, phase transition takes place at M_{ew} , releasing the background radiation. This radiation thermalizes the universe upto the

temperature $T_{\text{ew}} = 9.1 \times 10^{14} K$. Here, the event of phase transition is recognized as *big-bang*, which heralds our dynamical universe having the initial temperature $T_{\text{ew}} = 9.1 \times 10^{14} K$ and initial value of dark energy density $\rho_{\Lambda_{\text{ew}}} = 10^6 \text{GeV}^4$.

Moreover, it is important to mention that existence of *riccions* are possible at energy scales where higher-derivative terms in the gravitational action (2.1a) has significant role compared to Einstein-Hilbert term. At energy scales below M_{ew} , Einstein-Hilbert term dominates higher-derivative terms in the action (2.1a). So, *riccions* has no direct role in the evolution of proposed model of the universe, but it has two very important contributions to the observable universe as value of dark energy density $\rho_{\Lambda_{\text{ew}}} = 10^6 \text{GeV}^4$ and cosmic background radiation temperature $T_{\text{ew}} = 9.1 \times 10^{14} K$ at cosmic time $t = 0$.

Astronomical observations have compelling evidences that the current universe is dominated by dark energy(DE). The present cosmic dark energy density is very low, but it used to be very high in the early universe. The fall of dark energy density from very high to extremely low value can be explained if it is time-dependent. So, like other other cosmic dark energy models, here also, dark energy density ρ_{Λ} is slowly varying function of time. Following Bronstein's idea [14, 15, 16] that DE could decay to hot or cold dark matter, here it is proposed that $\rho_{\Lambda}(t)$ decays to hot dark matter(HDM) till temperature is high and cold dark matter after decoupling of matter from radiation. As a result, $\rho_{\Lambda}(t)$ falls from high value 10^6GeV^4 to currently low value $0.73\rho_{\text{cr},0}$ (where $\rho_{\text{cr},0}$ is the critical density).

It is proposed that the homogeneous dynamical universe begins with topology having the distance function

$$dS^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2] \quad (6.1)$$

for the spatially flat model of the universe supported by recent experiments [1, 2, 3, 34]. This space-time is a special case of hypersurface M^4 of the line element (1.1). Here $a(t)$ is the scale factor.

In what follows, it is obtained that *dark energy density* decreases with time from its initial value 10^6GeV^4 and falls down by 53 orders in the present universe. In 1933, Bronstein proposed that, in the expanding universe, *dark energy density* decreases due its decay as a result of emission of dark matter or radiation [15, 16]. The radiation, so emitted, could disturb spectrum of 3K-microwave background radiation. So, Bronstein's original idea was modified and it was introduced that dark energy (DE) could decay to hot or cold dark matter without any harm to spectrum of 3K-microwave background radiation [15]. Following this idea, here, it is demonstrated that *dark energy* decays to *dark matter* providing a solution to *cosmic coincidence problem*.

It is shown above that , in the beginning, the universe was very hot due to the background radiation (released during phase transition). Radiation energy density falls as

$$\rho_r = \frac{\rho_{\text{ew}}(r)a_{\text{ew}}^4}{a^4(t)}. \quad (6.2)$$

with growing scale factor.

Matter remains in thermal equilibrium with the background radiation for sufficiently long time. According to WMAP [37], decoupling of matter from background radiation takes place at $t_d \simeq 386 \text{kyr} = 1.85 \times 10^{37} \text{GeV}^{-1}$. Equation of state for radiation is $\omega = 1/3$. So, here,

it is proposed that *dark energy* decays to HDM till decoupling time t_d obeying $\omega_m = 1/3$.

When $t > t_d$, it decays more to CDM (which is non-baryonic and pressureless) with $\omega_m = 0$ as well as HDM. So, ratio of densities of HDM and CDM is extremely small below t_d .

In what follows, development of the universe is probed using these ideas.

(a) Decay of dark energy to dark matter

The conservation equations

$$T_{(\Lambda)j;i}^i + T_{(\text{dm})j;i}^i = 0 \quad (6.3)$$

yield coupled equations

$$\dot{\rho}_\Lambda + 3H(1 + \omega_\Lambda)\rho_\Lambda = -Q(t) \quad (6.4a)$$

and

$$\dot{\rho}_{\text{dm}} + 3H(1 + \omega_{\text{dm}})\rho_{\text{dm}} = Q(t) \quad (6.4b)$$

with $\rho_\Lambda(t)$ and $\rho_{\text{dm}}(t)$, being *dark energy* density and *dark matter* density respectively at cosmic time t . Here $H = \dot{a}/a$, $\omega_\Lambda = p_\Lambda/\rho_\Lambda = -0.4$ and $\omega_{\text{dm}} = p_{\text{dm}}/\rho_{\text{dm}}$. $Q(t)$ is the loss (gain) term for DE(DM) respectively.

Batchelor [28] has pointed out that dissipation is natural for all material fluids, barring superfluids. So, presence of a dissipative term for dark matter DM is reasonable. In eq.(6.4b), this term is obtained by taking

$$Q(t) = 3nH\rho_{\text{dm}} \quad (6.5a)$$

(with n being the real number) as in [29] without any harm to physics. With this setting for time-dependent arbitrary function $Q(t)$, eq.(6.4b) is obtained as

$$\dot{\rho}_{\text{dm}} + 3H(1 - n + w)\rho_{\text{dm}} = 0. \quad (6.5b)$$

Here $-n\rho_{\text{dm}}$ given by $Q(t)$ acts as dissipative pressure for DM. As $Q(t)$ is proportional to ρ_{dm} , this setting does not disturb the perfect fluid structure as required by the ‘cosmological principle’. Thus eq.(6.5b) yields the effective pressure for DM as

$$p_{(\text{dm},\text{eff})} = (-n + w)\rho_{\text{dm}}. \quad (6.5c)$$

Now connecting eqs.(6.4b) and (6.5) and integrating, it is obtained that

$$\rho_{\text{dm}} = Aa^{3(n-\omega_{\text{dm}}-1)} \quad (6.6a)$$

with

$$A = 0.23\rho_{\text{cr},0}a_0^{-3(n-1)} \quad (6.6b)$$

using current value of *dark matter* density $\rho_{\text{dm},0} = 0.23\rho_{\text{cr},0}$ and $a_0 = a(t_0)$.

Connecting eqs.(6.4a), (6.5) and (6.6a) and integrating, it is obtained that

$$\rho_{\Lambda} = \frac{D}{a^{3(1+\omega_{\Lambda})}} - \frac{n}{n + \omega_{\Lambda} - \omega_{\text{dm}}} \rho_{\text{dm}} \quad (6.7a)$$

where

$$D = 10^6 a_{\text{ew}}^{3(1+\omega_{\Lambda})} \text{GeV}^4 \quad (6.7b)$$

using the initial condition $\rho_{\text{dm}} = 0$ at $t = 0$.

(b) Expansion of the universe, when $\rho_{\Lambda} > \rho_{\text{dm}}$

Friedmann equation is given as

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}(\rho_{\Lambda} + \rho_{\text{dm}} + \rho_r) \simeq \frac{8\pi G_N}{3}(\rho_{\Lambda} + \rho_{\text{dm}}) \quad (6.8)$$

as $(\rho_{\Lambda} + \rho_{\text{dm}})$ dominates over ρ_r , which is clear from eqs.(6.2),(6.6) and (6.7).

In this case, the equation (6.8) reduces to

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 \simeq \frac{8\pi G_N D}{3a^{3(1+\omega_{\Lambda})}} \quad (6.9)$$

yielding the solution

$$\begin{aligned} a(t) &= a_{\text{ew}} \left[1 + \sqrt{\frac{8\pi G_N D}{3}} \frac{t}{a_{\text{ew}}^{9/10}} \right]^{10/9} \\ &= a_{\text{ew}} \left[1 + 2.89 \times 10^{-16} t \right]^{10/9} \end{aligned} \quad (6.10a)$$

with $\omega_{\Lambda} = -0.4$, $a_{\text{ew}} = a(t = 0)$ and

$$\sqrt{\frac{8\pi G_N D}{3}} = 2.986 \times 10^{-16} a_{\text{ew}}^{9/10} \text{GeV} \quad (6.10b)$$

($G_N = M_P^{-2}$, $M_P = 10^{19} \text{GeV}$).

It shows an accelerated expansion of the universe from the beginning itself as $\ddot{a}/a > 0$.

Connecting eqs.(6.7) and (6.10)

$$\rho_{\Lambda} = \frac{10^6}{\left[1 + \sqrt{\frac{8\pi G_N D}{3}} \left(\frac{t}{a_{\text{ew}}^{0.9}} \right) \right]^2} - \frac{n}{n + \omega_{\Lambda} - \omega_m} \rho_{\text{dm}}. \quad (6.11)$$

But, the present universe obeys the condition

$$\Omega_{\Lambda,0} + \Omega_{m,0} = 1, \quad (6.12a)$$

where $\Omega_{m,0} = \Omega_{dm,0} + \Omega_{r,0}$. Here $\Omega_0 = \rho_0/\rho_{cr,0}$ and $\rho_{cr,0} = 3H_0^2/8\pi G_N$ (H_0 being Hubble's constant) for the present universe. With these values the present *critical density* is calculated as

$$\rho_{cr,0} = 3H_0^2/8\pi G_N \simeq 1.2 \times 10^{-47} \text{GeV}^4 \quad (6.12b)$$

using $H_0 = h_0/t_0$ with $h_0 = 0.68$ and the present age of the universe $t_0 = 13.7 \text{Gyr} = 6.6 \times 10^{41} \text{GeV}^{-1}$ [37].

As, in the present universe, ρ_m is dominated by *cold dark matter* density, $\omega_m = 0$ is taken in eq.(6.11). Now, connecting eqs.(6.11) and (6.12), it is obtained that

$$n = 0.47 \quad (6.13)$$

Using eq.(6.10), the scale factor a_d at the decoupling time $t_d = 1.85 \times 10^{37} \text{GeV}^{-1}$ and the present scale factor a_0 are obtained as

$$a_d \simeq 1.38 \times 10^{24} a_{ew} \quad (6.14)$$

and

$$a_0 \simeq 1.58 \times 10^{29} a_{ew} \quad (6.15)$$

using the present age of the universe given above.

The equation(6.10) shows an accelerated growth of the scale factor when $\rho_{\Lambda} > \rho_{dm}$ exhibiting non-adiabatic expansion of the universe.

It implies non-conservation of entropy of the universe, which is in contrast to the decelerated adiabatic expansion of SMU, when cosmological model is radiation-dominated or matter-dominated.

In what follows, scale factor dependence of temperature and entropy is obtained in an empirical manner, based on current temperature of the microwave background radiation $T_0 = 2.73K$, the initial temperature $T_{ew} = 78.5\text{GeV} = 9.1 \times 10^{14}K$ given by eq.(5.9) and $a_0 \simeq 1.58 \times 10^{34}a_{ew}$ given by eq.(6.15). Thus, the required empirical relation is obtained as

$$\left[T/T_{ew}\right]^{103/50} = \frac{a_{ew}}{a} \quad (6.16)$$

which yields the decoupling temperature as

$$T_d = T_{ew} \left[\frac{a_{ew}}{a_d}\right]^{50/103} = 1740K, \quad (6.17)$$

using eqs.(5.9) and (6.14). This value is much lower than $T_d = 3000K$, obtained in the standard big-bang cosmology. These drastic changes are due to dominance of the *dark energy* in the proposed model.

Using eq.(5.8b), entropy of the universe is calculated as

$$S = \frac{7\pi^2}{90}a^3T^3. \quad (6.18)$$

Current value of entropy S_0 is supposed to be 10^{87} . “How could so high entropy of the universe be generated?” is an old question. A solution to this problem was suggested in a seminal paper on inflationary model of the early universe by Guth [38] and modified version of the same by Linde [39] and Albrecht and Steinhardt [40]. Here, a different answer to this question is provided on the basis of results obtained above.

Connecting eq.(6.15) and eq.(6.18) and current vlues of entropy as well as temperature $T_0 = 2.73K$ of the universe, it is obtained that

$$a_{\text{ew}} = 0.25 \quad (6.19)$$

as

$$S_0 = 10^{87} = \frac{7\pi^2}{90} (1.58 \times 2.73 \times 10^{29} a_{\text{ew}})^3$$

Connecting eqs.(6.16) and (6.18) it is obtained that entropy grows with the scale factor $a(t)$ as

$$S = \frac{7\pi^2}{90} T_{\text{ew}}^3 a_{\text{ew}}^{150/103} a^{159/103} \quad (6.20a)$$

with the initial value

$$S_{\text{ew}} = 9 \times 10^{42} \quad (6.20b)$$

The equation (6.6) yields the rate of production of HDM

$$\dot{\rho}_{\text{hdm}} = -5.9 \times 10^{-64} \frac{a_0^{1.59}}{a^{3.48}} \quad (6.21)$$

using $\omega_{\text{dm}} = 1/3$ for *hot dark matter*. This equation shows that rate of production of HDM increases with growing scale factor. So, production of HDM is responsible for increasing entropy in the universe. In SMU, number of photons decide entropy of the universe. But, in PM, photons and HDM both are responsible for entropy. As energy of HDM increases due to its production, owing to decay of DE, entropy is generated in this model. It is unlike SMU, where entropy remains conserved.

Connecting eqs.(6.6) and (6.16), temperature dependence of ρ_{dm} is obtained as

$$\rho_{\text{dm}} = 0.23\rho_{\text{cr},0} \frac{a_{\text{ew}}^{3(n-\omega_{\text{dm}}-1)}}{a_0^{3(n-1)}} \left[T_{\text{ew}}/T \right]^{6.18(n-\omega_{\text{dm}}-1)}. \quad (6.22)$$

Putting $\omega_{\text{dm}} = 1/3(0)$ for HDM (CDM), in eq.(6.22), ratio of CDM and HDM densities, ρ_{cdm} and ρ_{hdm} , is obtained as

$$\rho_{\text{cdm}}/\rho_{\text{hdm}} = a_{\text{ew}} \left[T_{\text{ew}}/T \right]^{2.06} \quad (6.23)$$

Eq.(6.23) shows that creation of HDM is higher at high temperature. But as temperature falls down, creation of CDM supersedes the production of HDM (production of *dark matter* owing to decay of *dark energy*). For the current universe, the ratio of eq.(6.23) is obtained as

$$\rho_{\text{cdm},0}/\rho_{\text{hdm},0} = 2.1 \times 10^{29} \quad (6.24)$$

using numerical values of a_{ew} , T_{ew} and T_0 (given above) in eq.(6.23). It shows that currently, ρ_{hdm} is almost negligible compared to ρ_{cdm} .

The scale factor a_{sd} , upto which *dark energy* vanishes due to decay, are obtained as

$$a_{\text{sd}} = 6.78a_0 \quad (6.25)$$

connecting eqs.(6.6),(6.7) and (6.10) as well as using $\rho_{\Lambda} = 0$. Here $\omega_{\text{dm}} = 0$ is taken as for $t > t_0$, *dark matter* content is expected to be dominated by CDM.

The ratio of densities of *dark energy*, ρ_{Λ} , and *dark matter*, ρ_{dm} , is obtained as

$$\rho_{\Lambda}/\rho_{\text{dm}} = 10 \left(a_0/a \right)^{0.21} - \frac{47}{7}. \quad (6.26)$$

It is interesting to note, from eq.(6.26), that the gap between *dark energy* density and *dark matter* density decreases with the growing scale factor. Connecting eqs.(6.6) and (6.7) and using n from eq.(6.13). This equation shows that $\rho_{\text{dm}}/\rho_{\Lambda} < 1$ for

$$a_{\text{ew}} < a < 3.44a_0 \quad \text{and} \quad 0 < t < 3.03t_0. \quad (6.27a, b)$$

The result, given by eq.(6.27b), solves the *cosmic coincidence problem* [24]. This approach, for solution of *coincidence problem* has an advantage that no scalar field is required to represent the *dark energy*, rather it is the contribution of higher-dimensional higher-derivative gravity to the observable universe. It is unlike the case of work in refs.[25-28], where assumed *quintessence* scalars are used.

(c) The universe, in case $\rho_{\Lambda} < \rho_{\text{dm}}$

Like eq.(6.27), employing the same procedure, it is also possible to find that $\rho_{\text{dm}}/\rho_{\Lambda} \geq 1$ for

$$a \geq 3.44a_0 \quad \text{and} \quad t \geq 3.03t_0. \quad (6.28a, b)$$

So, for $t \geq 3.03t_0$, the the Friedmann equation is written as

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}(\rho_{\Lambda} + \rho_{\text{dm}} + \rho_r) \simeq \frac{8\pi G_N}{3}\rho_{\text{dm}} \simeq 0.27H_0^2 \left(a_0/a\right)^{1.74} \quad (6.29)$$

using eqs.(6.6),(6.8), (6.13) and definition of $\rho_{\text{cr},0}$ (given above). Eq.(6.29) yields the solution

$$a(t) = \left[a_{\Lambda < \text{dm}}^{87/100} + 0.43 \times 10^{-43}(t - t_{\Lambda < \text{dm}}) \right]^{100/87}, \quad (6.30)$$

where $a_{\Lambda < \text{dm}} = 3.44a_0$ and $t_{\Lambda < \text{dm}} = 3.03t_0$. Eq.(6.30) also shows an accelerated growth of the scale factor, even though $\rho_{\Lambda} < \rho_{\text{dm}}$. It is

interesting to see that this expansion is faster than the expansion in the interval $0 < t < 3.03t_0$.

From eq.(6.25), the scale factor upto which *dark energy* vanishes is $a_{\text{sd}} = 430a_0$. So, eq.(6.30) yields the corresponding time as

$$t_{\text{sd}} = 3.7t_0 = 50.69\text{Gyr.} \quad (6.31)$$

It shows that decay of *dark energy* will continue in the interval $3.03t_0 < t < 3.7t_0$ also. So, during this time interval, the *dark matter* will follow the rule, given by eq.(6.6).

Using eqs.(6.16),(6.18), (6.25)and (6.31), T_{sd} and S_{sd} are calculated as

$$T_{\text{sd}} = T_0 \left(\frac{a_0}{a_{\text{sd}}} \right)^{50/103} = 0.395T_0 = 1.08K \quad (6.32)$$

Dark matter density at $t = t_{\text{sd}}$ is obtained as

$$\rho_{\text{dm(sd)}} = 1.32 \times 10^{-49} \text{GeV}^4 \quad (6.33)$$

using a_0 and a_{sd} in eq.(6.6).

As *dark energy* vanishes when $t \geq t_{\text{sd}}$, content of the universe will be dominated by CDM ,which is pressureless non-baryonic matter obeying the conservation equation

$$\dot{\rho}_{\text{dm}} + 3H\rho_{\text{dm}} = 0.$$

This equation yields scale factor dependence of ρ_{dm} as

$$\rho_{\text{dm}} = \rho_{\text{dm(sd)}} \left(a_{\text{sd}}/a(t) \right)^3 = \frac{2.54 \times 10^{39}}{a^3(t)} \quad (6.34)$$

for $t > 3.7t_0$.

Beyond the age of the universe $3.7t_0$, the Friedmann equation looks like

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}\rho_{\text{dm}} = \frac{212.8}{a^3(t)} \quad (6.35)$$

using eq.(6.34).

Eq.(6.35) yields the solution

$$a(t) = \left[a_{\text{sd}}^{3/2} \pm 14.6(t - t_{\text{sd}}) \right]^{2/3}, \quad (6.36)$$

with a_{sd} and t_{sd} given by eqs.(6.25) and (6.31) respectively.

On taking (+) sign in eq.(6.36), decelerated expansion of the universe is obtained beyond $t > 3.7t_0$ continuing for ever. But the (-) sign, in eq.(6.36), exhibits a contracting universe beyond $t > 3.7t_0$. So, ultimately the contracting universe is expected to collapse to a very small size with scale factor, possibly equal to $a_{\text{ew}} = 0.25$. From eq.(6.36), with (-) sign, the collapse time is calculated as

$$t_{\text{col}} = t_{\text{sd}} + \frac{a_{\text{ew}}^{3/2}}{14.6} = 18.08t_0 = 247.73\text{Gyr} \quad (6.37)$$

using $a_{\text{sd}} = 6.78a_0$ and $t_{\text{sd}} = 3.7t_0$.

7. Elementary particles, Primordial nucleosynthesis and Structure formation

7.1 Creation of particles

In SMU, it is assumed that elementary particles such as leptons, mesons, nucleons and their anti-particles were produced at the epoch of big-bang. Here, production of scalar and spin-1/2 particles is proposed in the very early universe due to topological changes caused by expansion of the universe.

A lot of work has been done in the past, where production of particles are discussed in curved space-time. This mechanism is based on the fact that, in the flat space-time, vacuum state in the Fock space is stable, if there is no source term in the action of the matter field. In curved space-time, vacuum is unstable even in the absence of the source term in the action. It turns out that gravity makes the initial vacuum state $|0, \text{in}\rangle$ unstable such that $|0, \text{in}\rangle \neq |0, \text{out}\rangle$, where $|0, \text{out}\rangle$ is the other vacuum state in the new Fock space. The instability of vacuum state shows production of particles where conformal symmetry is broken. Moreover, changing gravitational field contributes rest mass to produced particles [22, 32, 41]. Here $|0, \text{in}\rangle$ state is defined when $t \rightarrow 0$ and $|0, \text{out}\rangle$ is obtained for $t \rightarrow \infty$.

(a) Creation of spinless particles

The scalar field ϕ obeys the Klein-Gordon equation

$$(\square + m_\phi^2)\phi = 0, \quad (7.1)$$

where m_ϕ is mass of ϕ . Expanding ϕ in terms of mode k , eq.(7.1) is written as

$$\begin{aligned} (1 + 2.89 \times 10^{-16}t)\ddot{\phi}_k + \frac{28.9}{3} \times 10^{-16}\dot{\phi}_k + \left[m_{\phi_k}^2 (1 + 2.89 \times 10^{-16}t) \right. \\ \left. - \frac{k^2}{a_{\text{ew}}^2} (1 + 9.86 \times 10^{-16}t)^{-11/9} \right] \phi_k = 0 \end{aligned} \quad (7.3)$$

using the scale factor $a(t)$, given by eq.(6.10a).

For small t , eq.(7.3) is obtained as

$$\tau \frac{d^2 \phi_k^{\text{in}}}{d\tau^2} + \frac{10}{3} \frac{d\phi_k^{\text{in}}}{d\tau} + 10^{31} \left[-\frac{20k^2}{9a_{\text{ew}}^2} + \left(m_\phi^2 + \frac{11k^2}{9a_{\text{ew}}^2} \right) \tau \right] \phi_k^{\text{in}} = 0, \quad (7.4a)$$

where

$$\tau = 1 + 2.89 \times 10^{-16}t. \quad (7.4b)$$

Eq.(7.4) yields the normalized solution

$$\begin{aligned} \phi_k^{\text{in}} &= [2(2\pi)^3 \sqrt{b_2}]^{-1/2} e_1^{-i\tau\sqrt{b_2}} F_1\left(\frac{5}{3} + i\frac{b_1}{2\sqrt{b_2}}, \frac{10}{3}, 2i\tau\sqrt{b_2}\right) \\ &\approx [2(2\pi)^3 \sqrt{b_2}]^{-1/2} e^{-i\tau\sqrt{b_2}} \left[1 + \frac{(-3b_1 + 10i\sqrt{b_2})\tau}{10}\right], \end{aligned} \quad (7.5a)$$

where

$$b_1 = -\frac{20}{9} \frac{(10^{15.5}k)^2}{a_{\text{ew}}^2} \quad (7.5b)$$

and

$$b_2 = 10^{30} \left(m_\phi^2 + \frac{11k^2}{9a_{\text{ew}}^2}\right). \quad (7.5c)$$

Here ${}_1F_1(a, b, c)$ is the confluent hypergeometric function.

For large t , eq.(7.3) reduces to

$$\tau \frac{d^2 \phi_k^{\text{out}}}{d\tau^2} + \frac{10}{3} \frac{d\phi_k^{\text{out}}}{d\tau} + (10^{15.5}m_\phi)^2 \tau \phi_k^{\text{out}} = 0, \quad (7.6)$$

which integrates to

$$\phi_k^{\text{out}} = \tau^{-7/6} \left[c_1 J_{-7/6}(A\tau) + c_2 Y_{-7/6}(A\tau) \right], \quad (7.7a)$$

where

$$A = 10^{15.5}m_\phi. \quad (7.7b)$$

and $J_p(x)$ and $Y_p(x)$ are Bessel's function of first and second kind respectively. For large x , Bessel's functions can be approximated as

$$J_p(x) \simeq \frac{\cos(x - \pi/4 - p\pi/2)}{\sqrt{\pi x/2}} \quad (7.7c)$$

and

$$Y_p(x) \simeq \frac{\sin(x - \pi/4 - p\pi/2)}{\sqrt{\pi x/2}}. \quad (7.7d)$$

Using these approximations, when τ is large, eq.(7.7a) looks like

$$\phi_k^{\text{out}} \simeq \frac{\tau^{-5/3}}{\sqrt{\pi A/2}} [c_1 \cos(A\tau + \frac{\pi}{3}) + c_2 \sin(A\tau + \frac{\pi}{3})] \quad (7.8)$$

Solutions (7.5a) and (7.8) yield number of produced spinless particles (for mode k) per unit volume as

$$|\beta_k|^2 = \frac{\tau^{-10/3}}{\pi^2 A^2 b_2} |X|^2 \neq 0, \quad (7.9a)$$

where

$$\begin{aligned} X = & \left[\left(-\frac{3b_1\sqrt{b_2}\tau}{10} - \frac{2}{3}\sqrt{b_2} \right) \cos(A\tau) - A\sqrt{b_2}\tau \sin(A\tau) \right] + i \left[\left\{ \sqrt{b_2}\tau \right. \right. \\ & \left. \left. - \frac{3b_1}{10} + \frac{5}{3} \left(1 - \frac{3b_1}{10} \right) \tau^{-1} \right\} \cos(A\tau) + \left(1 - \frac{3b_1}{10} \right) A \sin(A\tau) \right]. \end{aligned} \quad (7.9b)$$

β_k is defined in Appendix B. Non-zero $|\beta_k|^2$ shows creation of spinless particles.

(b) Creation of spin-1/2 particles

The spin-1/2 field ψ satisfies the Dirac equation

$$(i\gamma^\mu D_\mu - m_f)\psi = 0, \quad (7.10)$$

where m_f is mass of ψ , γ^μ are Dirac matrices in curved space-time and D_μ are covariant derivatives defined in Appendix B. ψ can be written as

$$\psi = \sum_{s=\pm 1} \sum_k (b_{k,s} \psi_{Ik,s} + d_{k,s}^\dagger \psi_{IIk,s}) \quad (7.11)$$

with $b_{k,s}$ and $d_{k,s}$, given in Appendix B. Eqs.(7.10) and (7.11) yield

$$(i\gamma^\mu D_\mu - m_f)\psi_{Ik,s} = 0, \quad (7.12a)$$

$$(i\gamma^\mu D_\mu - m_f)\psi_{IIk,s} = 0. \quad (7.12b)$$

Now using the operator $(-i\gamma^\mu D_\mu - m_f)$ from left of eqs.(7.12a) and (7.12b), it is obtained that

$$(\square + \frac{1}{4}R + m_f^2)\tilde{\psi} = 0, \quad (7.13)$$

where $\tilde{\psi} = \psi_{Ik,s}(\psi_{IIk,s})$.

Writing

$$\psi_{Ik,s} = f_{Ik,s}(t)e^{i\vec{k}\cdot\vec{x}}u_s$$

and

$$\psi_{IIk,s} = f_{IIk,s}(t)e^{i\vec{k}\cdot\vec{x}}\hat{u}_s,$$

eq.(7.13) is obtained as

$$\tau \frac{d^2 \tilde{f}}{d\tau^2} + \frac{10}{3} \frac{d\tilde{f}}{d\tau} + 10^{30} \left[m_f^2 \tau - \frac{k^2}{\tau^{11/9}} - \frac{4.93 \times 10^{-15}}{3\tau} \right] \tilde{f} = 0, \quad (7.14)$$

where $\tilde{f} = f_{Ik,s}(f_{IIk,s})$.

For small t , eq.(7.14) looks like

$$\tau \frac{d^2 \tilde{f}}{d\tau^2} + \frac{10}{3} \frac{d\tilde{f}}{d\tau} + 10^{30} \left[(m_f^2 + \frac{11k^2}{9a_{\text{ew}}^2})\tau - \frac{20k^2}{9a_{\text{ew}}^2} \right] \tilde{f} = 0. \quad (7.15)$$

This equation is like the equation (7.4a). So, its solution has the same form. Using this solution in $\psi_{Ik,s}$ defined above, it is obtained that

$$\psi_{Ik,s}^{\text{in}} = \frac{e^{-i\tau\sqrt{b_2}}}{\sqrt{2(2\pi)^3\sqrt{b_2}}} \left[1 + \frac{(-3b_1 + 10i\sqrt{b_2})}{10}\tau \right] e^{i\vec{k}\cdot\vec{x}}u_s. \quad (7.16)$$

For large t , eq.(7.14) reduces to

$$\tau \frac{d^2 \tilde{f}}{d\tau^2} + \frac{10}{3} \frac{d\tilde{f}}{d\tau} + 10^{30} m_f^2 \tau \tilde{f} = 0 \quad (7.17)$$

yielding

$$\psi_{IIk,s}^{\text{out}} = \frac{\tau^{-5/3}}{\pi^2 A} \cos(A\tau) e^{i\vec{k} \cdot \vec{x}} \hat{u}_s. \quad (7.18)$$

Using $\psi_{Ik,s}^{\text{in}}$ and $\psi_{IIk,s}^{\text{out}}$ in the definition of $\beta_{k,s}$, given in the Appendix B,

$$\beta_{k,s} = (2\pi)^3 \tau^{-5/3} \frac{\cos(A\tau)}{\pi^2 A} \frac{e^{-i\tau\sqrt{b_2}}}{\sqrt{2(2\pi)^3 \sqrt{b_2}}} \left[1 + \frac{(-3b_1 + 10i\sqrt{b_2})}{10} \tau \right]. \quad (7.19)$$

So, the number of created spin-1/2 particles per unit volume is obtained from eq.(7.19) as

$$|\beta_{k,s}|^2 = \frac{(2\pi)^2}{2\pi^2 A \sqrt{b_2}} \tau^{-10/3} \left[\left(1 - \frac{3b_1}{10} \tau \right)^2 + b_2 \tau^2 \right]. \quad (7.20)$$

Eqs.(7.9a,b) and (7.20) show creation of spinless and spin-1/2 particles due to changing gravitational field in the proposed speeded-up model. The scale factor, given by eq.(6.10a) show that the significant change in the gravitational field is possible, in this model, when

$$t > 3.46 \times 10^{15.5} \text{GeV}^{-1} = 7.2 \times 10^{-9} \text{sec}. \quad (7.21)$$

as $a(t)$ remains almost constant upto this epoch. So, here, particle production is expected, when the universe is around $2.3 \times 10^{-9} \text{sec}$. old. Also, these results show that particle production falls down rapidly as time increases. Thus, like other models, here also particle production is expected in the early stages of the universe.

7.2 Primordial Nucleosynthesis

Hydrogen (H) is the major component of baryonic matter in the universe. The next main component is Helium-4 (4He). Occurrence of other light elements and metals is very small. It is found unlikely that abundance of 4He , Deuterium (D) and other light elements, in the universe, being caused by burn out of H in stars [42, 43] So, it is argued that the required amount of these might have produced in the early universe.

At the epoch of its formation, helium production depends upon neutron (n) concentration, which is determined by weak interaction reactions given as

$$n + \nu \rightleftharpoons p + e^-, n + e^+ \rightleftharpoons p + \bar{\nu} \quad (7.22)$$

where p stands for proton and ν for neutrino. This chemical equilibrium is maintained till weak reaction rate $\Gamma_w \gg H$, where H is the expansion rate of the universe given by eq.(6.9) and $\Gamma_w \simeq 1.3G_F^2 T^5$ with Fermi constant $G_F = \pi\alpha_w/\sqrt{2}M_w^2 = 1.17 \times 10^{-5} \text{GeV}^{-2}$. With the expansion of the universe, temperature decreases so weak interaction rate slows down. As a result, at the freeze-out temperature T_* ,

$$\Gamma_w \simeq H. \quad (7.23a)$$

Connecting eqs.(6.7b), (6.9), (6.16) and (7.23a), it is obtained that

$$1.3 \times (1.17)^2 \times 10^{-10} T_*^5 = 2.89 \times 10^{-16} \left(\frac{T_*}{T_{\text{ew}}} \right)^{1.9},$$

which yields

$$T_* = 0.9 \text{MeV}. \quad (7.23b)$$

Eqs.(6.10a,b) and (6.16) yield time dependence of temperature, for this model, as

$$t_{\text{sec}} = \frac{4.5}{T_{\text{MeV}}^{1.9}}. \quad (7.24)$$

Using this result, the freeze-out time t_* , corresponding to T_* , is obtained as

$$t_* \simeq 5.55 \text{ sec}. \quad (7.25)$$

In SMU, $T_* \simeq 0.86 \text{ MeV}$ and $t_* \simeq 1 \text{ sec}$ [44]. It is obtained that, in PM, T_* is a bit higher and freeze-out takes place later. The reason for these differences is the basic difference in developement of these models. SMU is driven by elementary particles present in the early universe and it expands adiabatically. The proposed model is driven by dark energy, with very high density at the beginning and it expands with acceleration.

Like [45], in this subsection, temperature T_9 is measured in units $10^9 K$, so 1 MeV becomes 11.6 in T_9 temperature. Now, eq.(7.24) looks as

$$t = \frac{474}{T_9^{1.9}}. \quad (7.26)$$

For large temperature $T_9 > 10$, neutron abundance is given by

$$X_n = \left[1 + e^{Q_9/T_9} \right]^{-1}, \quad (7.27a)$$

where $Q_9 = m_n - m_p = 15$. At freeze-out temperature $T_* \simeq 0.9 \text{ MeV}$, which is equivalent to $T_9 = 10.44$,

$$X_n^* = 0.19. \quad (7.27b)$$

Due to slightly higher freeze-out temperature, in the present model, neutron concentration is obtained higher than $X_n^* = 0.16$ in SMU.

When $\Gamma_w \ll H$ i.e. when chemical equilibrium between n and p freeze-out, neutron concentration is determined through neutron-decay

$$n \rightarrow p + e^- + \bar{\nu}. \quad (7.28)$$

Thus, for $t > t_*$, neutron abundance is given as

$$X_n(t) = 0.19e^{-t/\tau}, \quad (7.29)$$

where $\tau = (885.7 \pm 0.8)\text{sec.}$ is the neutron life-time.

First step, in the formation of complex nuclei, is

$$p + n \leftrightarrow D + \gamma$$

showing chemical equilibrium between deuteron (D) , nucleons and photon (γ). This equilibrium gives deuteron abundance by weight [43] as

$$X_D = 0.2 \times 10^{-12} (\Omega_B h^2) X_n X_p T_9^{3/2} \exp(B_D/T_9), \quad (7.30)$$

where $X_D = 2n_D/n_B$, $X_n = n_n/n_B$, $X_p = n_p/n_B$, $X_n + X_p = 1$, deuterium binding energy $B_D = m_p + m_n - m_D = 2.23\text{MeV}$ which yields $B_{D9} = 25.82$ (in units of T_9) and $(\Omega_B h^2)$ is the baryon number density.

Light heavy elements ${}^4\text{He}$ is formed through reactions

$$n + p \rightarrow D + \gamma, \quad (7.31a)$$

$$(i) D + D \rightarrow {}^3\text{He} + n, (ii) D + D \rightarrow {}^3\text{H} + p \quad (7.31b, c)$$

and

$$(ii) D + {}^3\text{H} \rightarrow {}^4\text{He} + n, (ii) {}^3\text{He} + D \rightarrow {}^4\text{He} + p, \quad (7.31d, e)$$

where ${}^3\text{H}$ stands for tritium.

These reactions show that sufficient deuterium abundance is required for nucleosynthesis of ${}^4\text{He}$, which is supposed to be around 25% of the baryonic content of the universe. But after weak-interaction freeze out, deuterium abundance is not sufficient unless temperature is very low. For example, at $T_9 = 5.8$ ($T = 0.5\text{MeV}$), $X_D = 2 \times 10^{-12}$ is obtained using eq.(7.30) and putting $\Omega_B h^2 = 0.05$.

Rates for reactions (7.30b,c) are given as [45, Appendix]

$$\begin{aligned}\lambda_{DD(i)} &= 3.97 \times 10^5 T_9^{-2/3} e^{-4.258/T_9^{1/3}} [1 + 0.098 T_9^{1/3} + 0.876 T_9^{2/3} \\ &\quad + 0.6 T_9 - 0.041 T_9^{4/3} - 0.071 T_9^{5/3}] (\Omega_B h^2) \text{sec}^{-1} \\ \lambda_{DD(ii)} &= 4.17 \times 10^5 T_9^{-2/3} e^{-4.258/T_9^{1/3}} [1 + 0.098 T_9^{1/3} + 0.518 T_9^{2/3} \\ &\quad - 0.355 T_9 - 0.010 T_9^{4/3} - 0.018 T_9^{5/3}] (\Omega_B h^2) \text{sec}^{-1}\end{aligned}\tag{7.32a, b}$$

Making approximations in eqs.(7.30a,b), it is obtained that

$$\begin{aligned}\lambda_{DD} &= \lambda_{DD(i)} + \lambda_{DD(ii)} \\ &\sim 8.14 \times 10^5 T_9^{-2/3} e^{-4.258/T_9^{1/3}} (\Omega_B h^2) \text{sec}^{-1}\end{aligned}\tag{7.33}$$

Mukhanov [44] has obtained the condition for conversion of sufficient amount of D to ${}^3\text{He}$ and ${}^3\text{H}$ as

$$\frac{1}{2} \lambda_{DD} X_D t \sim 1\tag{7.34}$$

From eqs.(7.27), (7.30)-(7.33), it is obtained that

$$6.43 \times 10^{-6} T_{9i}^{-1.3} e^{\left(\frac{25.82}{T_{9i}} - \frac{4.258}{(T_{9i})^{1/3}}\right)} (\Omega_B h^2)^2 \sim 1\tag{7.35}$$

showing a relation between $\Omega_B h^2$ and T_{9i} . T_{9i} is the temperature at which nucleosynthesis takes place. For $\Omega_B h^2 = 0.05$, this equation

yields

$$T_{9i} \sim 1.16, \quad (7.36)$$

which corresponds to $T_i = 0.1\text{MeV}$. It shows that if baryon number density $\Omega_B h^2$ is ~ 0.05 , nucleosynthesis of ${}^4\text{He}$ may take place at temperature $T_i = 0.1\text{MeV}$. In SMU, this temperature is $\sim 0.086\text{MeV}$.

Eqs.(7.29) and (7.35) yield deuterium concentration at T_i as

$$X_{Di} \simeq 8.93 \times 10^{-6} \quad (7.37)$$

It happens at the cosmic time

$$t_i = 357.5\text{sec}. \quad (7.38)$$

Thus, it is obtained that, in the present model, nucleosynthesis of ${}^4\text{He}$ begins much later compared to SMU, where it happens at time $\sim 100\text{sec}$ [44]. But, it is still less than life-time of a neutron, which is required for the nucleosynthesis.

The final ${}^4\text{He}$ abundance is determined by available free neutrons at cosmic time t_i . As total weight of ${}^4\text{He}$ is due to neutron and proton, so its final abundance by weight is given by [45] as

$$X_\alpha^f = 2X_n^* \exp(-t^f/\tau) = 2X_n^* \exp(-0.535/(T_9^f)^{1.9}) \quad (7.39)$$

using eqs.(7.26) and $\tau = 886.5\text{sec}$. Here α stands for ${}^4\text{He}$.

The rate of production of ${}^4\text{He}$ is given by the equation

$$\dot{X}_\alpha = 2\lambda_{DD(ii)} X_D^2. \quad (7.40)$$

As free neutrons are captured in ${}^4\text{He}$, helium production dominates neutron decay. It happens when [45]

$$2\dot{X}_\alpha = \frac{X_n}{\tau}. \quad (7.41)$$

Approximating $\lambda_{DD(ii)}$, given by eq.(7.32b), around $T_9 = 1.16$, it is obtained that

$$\lambda_{DD(ii)} \simeq 6.98 \times 10^5 T_9^{-2/3} \exp(-4.258/T_9^{1/3})(\Omega_B h^2). \quad (7.42)$$

Eqs.(7.29), (7.38)-(7.40) yield

$$\begin{aligned} \frac{1}{2\tau} &\simeq 6.98 \times 10^5 T_9^{7/3} e^{(\frac{51.64}{T_9} - \frac{4.258}{T_9^{1/3}})} \times 0.19 e^{(-\frac{0.535}{T_9^{2.133}})} \\ &\times (0.2 \times 0.81 \times 10^{-12})^2 (\Omega_B h^2)^3, \end{aligned}$$

which implies that

$$T_9^f = \frac{51.64}{37.96 - \frac{7}{3} \ln T_9 - 3 \ln(\Omega_B h^2) + \frac{4.258}{T_9^{1/3}} + \frac{0.535}{T_9^{2.133}}}. \quad (7.43)$$

At $T = T_{9i} = 1.16$,

$$T_9^f = \frac{51.64}{42.08 - 3 \ln(\Omega_B h^2)}. \quad (7.44)$$

Using eq.(7.44) in eq.(7.39), helium abundance by weight, as function of baryon number density ($\Omega_B h^2$), is obtained as

$$X_\alpha^f \simeq 0.38 \exp \left[-0.535 \left\{ \frac{42.08 - 3 \ln(\Omega_B h^2)}{51.64} \right\}^{1.9} \right]. \quad (7.45)$$

For $\Omega_B h^2 = 0.05$, it is obtained that

$$X_\alpha^f \simeq 0.23. \quad (7.46)$$

For $T < T_9^f$, neutron abundance is governed by $\dot{X}_n = -2\dot{X}_\alpha$ with \dot{X}_α given by eq.(7.39). So,

$$\dot{X}_n = -2\lambda_{DD(ii)} X_D^2,$$

where X_D and $\lambda_{DD(ii)}$ are given by eqs.(7.30) and (7.41) respectively. Using eq.(7.26) and integrating, this equation yields neutron abundance below T_9^f as

$$X_n = \left[(1/X_n^\alpha) + (\Omega_B h^2)^3 e^{-37.62} T_9^{0.43} \left(e^{51.64/T_9} - e^{51.64/T_9^f} \right) \right]^{-1}, \quad (7.47a)$$

which shows that concentration of free neutrons drops very fast. It is due to capture of free neutrons in 4He . From eq.(7.29), it is obtained that neutron and deuterium concentrations become equal at temperature T_9^d given as [45]

$$T_9^d = \frac{25.82}{29.45 - \ln(\Omega_B h^2) - \frac{3}{2} \ln T_9^d}. \quad (7.47b)$$

Deuteron concentration is given by [45]

$$\dot{X}_D \simeq -2\lambda_{DD(ii)} X_D^2 = -2 \times 8.14 \times 10^5 T^{-2/3} (\Omega_B h^2) e^{-4.258/T_9^{1/3}} X_D^2,$$

which integrates to

$$X_D = \left[\left(1/X_D(T_9^d) \right) + (\Omega_B h^2) e^{20.16} \{ (T_9^d)^{-2.57} e^{-4.258/(T_9^d)^{1/3}} - (T_9)^{-2.8} e^{-4.258/(T_9)^{1/3}} \} \right]^{-1}, \quad (7.47c)$$

where $X_D(T_9^d) = X_n(T_9^d)$.

Eqs.(7.47a) and (7.47c) show that, below T_9^f , abundance of free neutron and deuterium drop rapidly in PM also like SMU.

7.3 Growth of inhomogeneities in the proposed model

For small inhomogeneities, linear perturbation of Einstein equations is enough. In this case, contrast density $\delta = \delta\rho_{\text{dm}}/\rho_{\text{dm}}$ (with ρ_{dm} , being the cold dark matter energy density and $\delta\rho_{\text{dm}}$ is small fluctuation in

ρ_{dm} presenting inhomogeneity), obeys a linear differential equation in the homogeneous model of the universe [46]. Due to linearity, δ can be expanded in modes k as

$$\delta = \sum_k \delta_k e^{i\vec{k} \cdot \vec{x}}. \quad (7.48)$$

For modes k with proper wavelength $F < H^{-1}(t)(H^{-1}(t)$ being the Hubble's radius), the perturbation equation looks like

$$\ddot{\delta}_k + 2\frac{\dot{a}}{a}\dot{\delta}_k + \left(\frac{k^2 v_s^2}{a^2} - 4\pi G \rho_{\text{dm}}\right)\delta_k = 0, \quad (7.49)$$

which is free from gauge ambiguities [46, eq.(4.158)]. So, δ_k depends on t only.

Eqs.(6.5c) and (6.13) give effective pressure for DM as

$$p_{(\text{dm},\text{eff})} = (-0.47 + w_{\text{dm}})\rho_{\text{dm}}. \quad (7.50)$$

It yields

$$v_s^2 = \frac{dp_{(\text{dm},\text{eff})}}{d\rho_{\text{dm}}} = -0.47 + w. \quad (7.51)$$

Connecting eqs.(6.6), (6.13), (7.49) and (7.51), the differential equation for δ_k is obtained as

$$\ddot{\delta}_k + 2\frac{\dot{a}}{a}\dot{\delta}_k + \left[(-0.47 + w_{\text{dm}})\frac{k^2}{a^2} - \left(0.345H_0^2 a_0^{1.59}/a^{3(0.53+w_{\text{dm}})}\right)\right]\delta_k = 0 \quad (7.52)$$

with $\rho_{\text{cr},0} = 3H_0^2/8\pi G$.

Using eq.(6.10), eq.(7.52) looks like

$$\tau \frac{d^2 \delta_k}{d\tau^2} + \frac{10}{3} \frac{d\delta_k}{d\tau} + 10^{31} \left[(-0.47 + w_{\text{dm}}) \frac{k^2}{\tau^{11/9}} - \frac{0.345H_0^2 a_0^{1.59}}{\tau^{10(0.23+w_{\text{dm}})/3}} \right] \delta_k = 0, \quad \text{eqno(7.53)}$$

where τ is defined in eq.(7.4b).

As mentioned above, for CDM $w_{\text{dm}} = 0$, so eq.(7.52) is approximated as

$$\tau \frac{d^2 \delta_k}{d\tau^2} + \frac{10}{3} \frac{d\delta_k}{d\tau} - \frac{3.45 \times 10^{29} H_0^2 a_0^{1.59}}{\tau^{2.3/3}} \delta_k = 0, \quad (7.54)$$

which is integrated to

$$\begin{aligned} \delta_k &= \tau^{-11/18} \left[c_1 H_{110/21}^{(1)}(2ib\tau^{7/60}) + c_2 H_{-110/21}^{(2)}(2ib\tau^{7/60}) \right] \\ &= \tau^{-7/6} \left[(c_1 + c_2) J_{110/21}(2ib\tau^{7/60}) + i(c_1 - c_2) Y_{110/21}(2ib\tau^{7/60}) \right], \end{aligned} \quad (7.55)$$

where $i = \sqrt{-1}$ and $b^2 = 3.45 \times 10^{30} H_0^2 a_0^{1.59}$. Here $H_p^{(1)}(x)$ and $H_p^{(2)}(x)$ are Hankel's functions as well as $J_p(x)$ and $Y_p(x)$ are Bessel's functions. Moreover c_1 and c_2 are integration constants.

As discussed in section 6, CDM dominates HDM in the late universe, so structure formation is expected for large τ . Using approximations (7.7c,d) for Bessel functions, for large τ in eq.(7.55) and doing some manipulations, it is obtained that

$$\delta_k \approx \tau^{-0.67} e^{b\tau^{7/60}}, \quad (7.56)$$

which shows growth of structure formation as cosmic time t increases.

8. Summary of Results

In contrast to the original *big – bang* theory, the proposed cosmology answers many basic questions as (i) “What is the *fireball*?”, (ii) “How does it burst out?”, (iii) “What is the background radiation?” and “What are initial values of temperature and energy density?” Moreover, the proposed cosmological model is free from initial singularity. It is found that the *dark energy* violates the *strong energy condition* showing “bounce” of the universe, which is consistent with

singularity – free model of cosmology [20, 24, 25]. The initial scale factor is computed to be $a_{\text{ew}} = 0.25$. It is in contrast to the standard model of cosmology SMU, which encounters with singularity having zero scale factor, infinite energy density and infinite temperature.

The present value of *dark energy* density is supposed to be $\sim 7.3 \times 10^{-48} \text{Gev}^4$, which is 53 orders below its initial values 10^6Gev^4 . The question “How does *dark energy* falls by 53 orders in the current universe?” is answered by the result, in section 6(a), showing that *dark energy* decays to *dark matter*, obeying the rule, given by eq.(6.7). Upto the decoupling time $t_d \simeq 386 \text{kyr}$, matter remains in thermal equilibrium with the background radiation, so produced *dark matter* upto t_d is supposed to be HDM. But when $t > t_d$, production of CDM is more than HDM. The current ratio of HDM density and CDM density is found to be 5×10^{-30} . In section 6, it is found that creation of HDM raises entropy of the universe upto $\sim 10^{87}$. During the phase of accelerated expansion, temperature falls as $a(t)^{-50/103}$.

It is found that one of the two types of dynamical changes of the universe are possible, beyond $3.7t_0 = 50.69 \text{Gyrs}$, (i) decelerated expansion and (ii) contraction. If future universe expands with deceleration, it will expand for ever. But in the case of contraction, it will collapse by 247.73Gyrs .

As *dark energy* dominates over *dark matter*, from the beginning upto the time $3.03t_0$, no *cosmic coincidence problem* arises in the proposed scenario. Moreover, in the preceding section, some other basic problems such as creation of particles in the early universe, primordial nucleosynthesis and structure formation in the late universe. In the

proposed model, it is shown that particles are created due to topological changes which is unlike the SMU, where it is assumed that elementary particles are created at the time of big-bang. It is also shown, in section 7.2, that process of primordial nucleosynthesis goes very well in the proposed model predicting $\sim 23\%$ Helium-4 abundance by weight. In section 7.3, it is shown that, in the late universe, inhomogeneities in CDM grow exponentially with time.

Thus the proposed model is able to provide possible solutions to many cosmological problems with prediction for the future universe.

Appendix A

Riccion and Graviton

From the action

$$S = \int d^4x d^Dy \sqrt{-g_{(4+D)}} \left[\frac{M^{(2+D)} R_{(4+D)}}{16\pi} + \alpha_{(4+D)} R_{(4+D)}^2 + \gamma_{(4+D)} (R_{(4+D)}^3 - \frac{6(D+3)}{D-2} \square_{(D+4)} R_{(D+4)}^2) \right], \quad (A.1)$$

the gravitational equations are obtained as

$$\frac{M^{(2+D)}}{16\pi} (R_{MN} - \frac{1}{2} g_{MN} R_{(4+D)}) + \alpha_{(4+D)} H_{MN}^{(1)} + \gamma_{(4+D)} H_{MN}^{(2)} = 0, \quad (A.2a)$$

where

$$H_{MN}^{(1)} = 2R_{;MN} - 2g_{MN} \square_{(4+D)} R_{(4+D)} - \frac{1}{2} g_{MN} R_{(4+D)}^2 + 2R_{(4+D)} R_{MN}, \quad (A.2b)$$

and

$$H_{MN}^{(2)} = 3R_{;MN}^2 - 3g_{MN} \square_{(4+D)} R_{(4+D)}^2 - \frac{6(D+3)}{(D-2)} \left\{ -\frac{1}{2} g_{MN} \square_{(4+D)} R_{(4+D)}^2 \right\}$$

$$+2\Box_{(4+D)}R_{(D+4)}R_{MN}+R_{;MN}^2\}-\frac{1}{2}g_{MN}R_{(4+D)}^3+3R_{(4+D)}^2R_{MN}. \quad (A.2c)$$

Taking $g_{MN} = \eta_{MN} + h_{MN}$ with η_{MN} being $(4 + D)$ -dimensional Minkowskian metric tensor components and h_{MN} as small fluctuations, the equation for *graviton* are obtained as

$$\Box_{(4+D)}h_{MN} = 0 \quad (A.3)$$

neglecting higher-orders of h .

On compactification of $M^4 \otimes S^D$ to M^4 , eq.(A.3) reduces to the equation for 4-dimensional *graviton* as

$$\Box h_{\mu\nu} + \frac{l(l+D-1)}{\rho^2}h_{\mu\nu} = 0 \quad (A.4)$$

for the space time

$$dS^2 = g_{\mu\nu}dx^\mu dx^\nu - \rho^2[d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \cdots + \sin^2\theta_1 \cdots \sin^2\theta_{(D-1)} d\theta_D^2]. \quad (A.5)$$

The 4-dimensional graviton equation (A.4) is like usual 4-dimensional graviton equation (the equation derived from 4-dimensional action) only for $l = 0$. Thus the massless graviton is obtained for $l = 0$ only.

As explained, in section 2, the trace of equations (A.2) leads to the *riccion* equation

$$[\Box + \frac{1}{2}\xi R + m_{\tilde{R}}^2 + \frac{\lambda}{3!}\tilde{R}^2]\tilde{R} + \vartheta = 0, \quad (A.6a)$$

where

$$\begin{aligned}
\xi &= \frac{D}{2(D+3)} + \eta^2 \lambda R_D \\
m_{\tilde{R}}^2 &= -\frac{(D+2)\lambda V_D}{16\pi G_{(4+D)}} + \frac{DR_D}{2(D+3)} + \frac{1}{2}\eta^2 \lambda R_D^2 \\
\lambda &= \frac{1}{4(D+3)\alpha}, \\
\vartheta &= -\eta \left[-\frac{(D+2)\lambda M^{(2+D)} V_D}{16\pi} + \frac{DR_D^2}{4(D+3)} + \frac{1}{6}\eta^2 \lambda R_D^3 \right].
\end{aligned}$$

The graviton $h_{\mu\nu}$ has 5 degrees of freedom (2 spin-2 graviton, 2 spin-1 gravi-vector (gravi-photon) and 1 scalar). The scalar mode f satisfies the equation

$$\square f + \frac{l(l+D-1)}{\rho^2} f = 0 \quad (\text{A.7})$$

Comparison of eqs.(A.6) and (A.7) show many differences between scalar mode f of graviton and the riccion (\tilde{R}) e.g. ξ, λ and ϑ , given by eqs.(A.6b,c,d,e), are vanishing for f , but non-vanishing for \tilde{R} . Eq.(A.7) shows $(mass)^2$ for f as

$$m_f^2 = \frac{l(l+D-1)}{\rho^2} f, \quad (\text{A.8})$$

whereas $(mass)^2$ for \tilde{R} , given by eq.(A.6c), depends on G_{4+D}, V_D and R_D (given in section 2). $m_f^2 = 0$ for $l = 0$, but $m_{\tilde{R}}^2$ can vanish only when gravity is probed upto $\sim 10^{-33}cm$. As mentioned above, so far, gravity is probed only upto 1cm. Thus $(mass)^2$ of riccion does not vanish.

So, even though, f and \tilde{R} are scalars arising from gravity, both are different. *Riccion* can not arise without higher-derivative curvature terms in the gravitational action, but *graviton* can be obtained even from Einstein-Hilbert action.

$$\square = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left(\sqrt{-g} g^{\mu\nu} \frac{\partial}{\partial x^\nu} \right) = \eta^{\mu\nu} \frac{\partial}{\partial X^\mu \partial X^\nu},$$

where X^μ are locally inertial co-ordinates and $\eta^{\mu\nu}$ are Minkowskian metric components. It shows that the scalar like operator \square has the same role on \tilde{R} as it is for other scalar fields ϕ due to principle of equivalence.

Appendix B

Bogoliubov transformations

In a Hilbert space, a scalar Φ satisfying the Klein-Gordon equation can be written as linear combinations

$$\begin{aligned} \Phi &= \sum_k \left[A_k^{\text{in}} \Phi_k^{\text{in}} + A_k^{\dagger \text{in}} \Phi_k^{*\text{in}} \right] \\ &= \sum_k \left[A_k^{\text{out}} \Phi_k^{\text{out}} + A_k^{\dagger \text{out}} \Phi_k^{*\text{out}} \right], \end{aligned} \tag{B.1a}$$

where Φ_k^{out} and Φ_k^{in} both belong to the same Hilbert space. As a result, one obtains

$$\Phi_{klm}^{\text{out}} = \alpha_k \Phi_k^{\text{in}} + \beta_k \Phi_k^{*\text{in}}, \tag{B.1b}$$

where α_k and β_{km} are Bogolubov coefficients satisfying the condition

$$|\alpha_k|^2 - |\beta_k|^2 = 1. \tag{B.2}$$

The in- and out- vacuum states are defined as

$$A_k^{\text{in}} |\text{in} \rangle = 0 = A_k^{\text{out}} |\text{out} \rangle. \tag{B.3a, b}$$

Moreover,

$$A_k^{\text{out}} = \alpha_k A_k^{\text{in}} + \beta_k A_k^{*\text{in}}, \tag{B.3c}$$

The normalization condition for Φ is given as

$$(\Phi_k, \Phi_k) = 1 = -(\Phi_k^*, \Phi_k^*), (\Phi_k, \Phi_k^*) = 0 \quad (B.4a, b, c)$$

where the scalar product is defined as

$$(\Phi_k, \Phi_k) = - \int \sqrt{-g_\Sigma} d\Sigma^\mu [\Phi_k (\partial_\mu \Phi_{k'l'm'}^*) - (\partial_\mu \Phi_k) \Phi_{k'l'm'}^*], \quad (B.4d)$$

where Σ is the 3-dim. hypersurface.

Connecting eqs.(B.1b) and (B.4a,b,c),

$$\alpha_k = (\Phi_k^{\text{out}}, \Phi_k^{\text{in}}) \quad (B.5a)$$

and

$$\beta_k = -(\Phi_k^{\text{out}}, \Phi_k^{*\text{in}}) \quad (B.5b)$$

(b) Spin-1/2 field

The spin-1/2 field ψ , satisfies the Dirac equation

$$(i\gamma^\mu D_\mu - m_f)\psi = 0, \quad (B.6a)$$

where m_f is mass of ψ and

$$D_\mu = \partial_\mu - \Gamma_\mu \quad (B.6b)$$

with Dirac matrices in curved space-time

$$\gamma^\mu = e_a^\mu \tilde{\gamma}^a, \quad (B.7a)$$

where $(a, \mu = 0, 1, 2, 3)$ and e_a^μ are defined through

$$e_a^\mu e_b^\nu g_{\mu\nu} = \eta_{ab}. \quad (B.7b)$$

Here η_{ab} are Minkowskian metric tensor components and $g_{\mu\nu}$ are metric tensor components in curved space-time. *c.c.* stands for complex conjugation.

Dirac matrices γ^μ in curved space-time satisfy the anti-commutation rule [22, 32]

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \quad (B.7c)$$

and Dirac matrices $\tilde{\gamma}^a$ in Minkowskian space-time satisfy the anti-commutation rule

$$\{\tilde{\gamma}^a, \tilde{\gamma}^b\} = 2\eta^{ab}. \quad (B.7d)$$

Γ_μ are defined as

$$\Gamma_\mu = -\frac{1}{4} \left(\partial_\mu e_a^\rho + \Gamma_{\sigma\mu}^\rho e_a^\sigma \right) g_{\nu\rho} e_b^\nu \tilde{\gamma}^b \tilde{\gamma}^a. \quad (B.7e)$$

Further, ψ can be decomposed as [22]

$$\begin{aligned} \psi &= \sum_{s=\pm 1} \sum_k \left(b_{k,s}^{\text{in}} \psi_{I(k,s)}^{\text{in}} + d_{-k,-s}^{\dagger \text{in}} \psi_{II(-k,-s)}^{\text{in}} \right) \\ &= \sum_{s=\pm 1} \sum_k \left(b_{k,s}^{\text{out}} \psi_{I(k,s)}^{\text{out}} + d_{-k,-s}^{\dagger \text{out}} \psi_{II(-k,-s)}^{\text{out}} \right), \end{aligned} \quad (B.8a, b)$$

as both in- and out-spinors belong to the same Hilbert space. The in- and out-vacuum states are defined as

$$b_{k,s}^{\text{in}} |\text{in}\rangle = d_{-k,s}^{\text{in}} |\text{in}\rangle = 0 \quad (B.9a, b)$$

and

$$b_{k,s}^{\text{out}} |\text{out}\rangle = d_{-k,s}^{\text{out}} |\text{out}\rangle = 0 \quad (B.9c, d)$$

Bogoliubov transformations are given as [22]

$$\begin{aligned} b_{k,s}^{\text{out}} &= b_{k,s}^{\text{in}} \alpha_{k,s} + d_{-k,-s}^{\dagger \text{in}} \beta_{k,s} \\ b_{k,s}^{\dagger \text{out}} &= \alpha_{k,s}^* b_{k,s}^{\dagger \text{in}} + \beta_{k,s}^* d_{-k,-s}^{\text{in}} \\ d_{-k,-s}^{\dagger \text{out}} &= b_{k,s}^{\text{in}} \alpha_{k,s} + d_{-k,-s}^{\dagger \text{in}} \beta_{k,s} \\ d_{-k,-s}^{\text{out}} &= \alpha_{k,s}^* b_{k,s}^{\dagger \text{in}} + \beta_{k,s}^* d_{-k,-s}^{\text{in}}. \end{aligned}$$

$$(B.10a, b, c, d)$$

Connecting eqs.(B.8)-(B.10), one obtains [33]

$$|\alpha_k|^2 + |\beta_k|^2 = \sum_s |\alpha_{k.s}|^2 + |\beta_{k.s}|^2 = 1, \quad (B.11)$$

where

$$\alpha_{k.s} = \int_{\Sigma} \sqrt{-g_{\Sigma}} d^3x \bar{\psi}_{I(k,s)}^{\text{out}} \tilde{\gamma}^0 \psi_{I(k,s)}^{\text{in}} \quad (B.12a)$$

and

$$\beta_{k.s} = \int_{\Sigma} \sqrt{-g_{\Sigma}} d^3x \bar{\psi}_{II(k,s)}^{\text{out}} \tilde{\gamma}^0 \psi_{II(k,s)}^{\text{in}} \quad (B.12b)$$

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